METAL-PLATE WORK.

BOOK I.

CHAPTER I.

CLASSIFICATION.

(1.) NOTWITHSTANDING the introduction of machinery and the division of labour in the various branches of metal-plate work, there is as great a demand for good metal-plate workers as ever, if not indeed a greater demand than formerly, while the opportunities for training such men are becoming fewer. An important part of the technical education of those connected with sheet-metal work is a knowledge of the setting-out of patterns. Such knowledge, requisite always by reason of the variety of shapes that are met with in articles made of sheet-metal, is nowadays especially needful; in that the number of articles made of sheet metal, through the revival of art metal-work, the general advance of science, and the introduction of new designs (which in many cases have been very successful), in articles of domestic use, has considerably increased. It is with the setting-out of patterns that this volume principally deals. To practical men, the advantages in saving of time and material, of having correct patterns to work from, are obvious. Whilst, however, the method of treatment here of the subject will be essentially practical, an amount of theory sufficient for a thorough comprehension of the rules given will be introduced, a knowledge of rules without principles being mere 'rule of thumb,' and not true technical education.

(2.) Starting in the following pages with some introductory problems and other matter, we shall proceed from these to the articles for which patterns are required by sheet-metal workers and which may be thus conveniently classed and subdivided:

CLASS I.—*Patterns for Articles of equal taper or inclination* (pails, oval teapots, gravy strainers, &c.).	Sub-divisions.	*a.* Of round surfaces. *b.* Of plane or flat surfaces. *c.* Of curved and plane surface combined.
CLASS II.—*Patterns for Articles of unequal taper or inclination* (baths, hoppers, canister-tops, &c.).	Sub-divisions.	*a.* Of round surfaces. *b.* Of plane or flat surfaces. *c.* Of curved and plane surface combined.

CLASS III.—*Patterns for Miscellaneous Articles* (elbows, and articles of compound bent surface, as vases, aquarium stands, mouldings, &c).

All these articles will be found dealt with in their several places.

We shall conclude with a few technical details in respect of the metals that metal-plate workers mostly make use of.

(3.) The setting out of patterns in sheet-metal work belongs to that department of solid geometry known as "Development of Surfaces," which may be said to be the spreading or laying out without rupture the surfaces of solids in the plane or flat, the plane now being sheet metal.

CHAPTER II.

INTRODUCTORY PROBLEMS; WITH APPLICATIONS.

DEFINITIONS.

Straight Line.—A straight line is the shortest distance between two points.

NOTE.—If not otherwise stated, lines are always supposed to be straight.

Angle.—An angle is the inclination of two lines, which meet, one to another. The lines A B, C B in Fig. 1 which are inclined to each other, and meet in B, are said to form an *angle* with one another. To express an angle, the letters which denote the two lines forming the angle are employed, the letter at the *angular point* being placed in the middle; thus, in Fig. 1, we speak of the angle A B C.

FIG. 1.

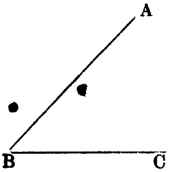


FIG. 2.

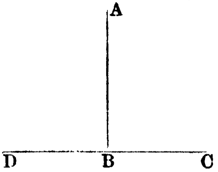
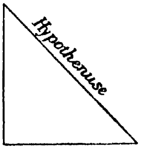


FIG. 3.



Perpendicular. Right Angles — If a straight line, A B (Fig. 2), meets or stands on another straight line, C D, so that the adjacent angles (or angles on either side of A B) A B D, A B C, are equal, then the line A B is said to be *perpendicular* to, or *at right angles* with ('square with') D C, and each of the angles is a right angle.

Parallel Lines.—Parallel lines are lines which, if produced ever so far both ways, do not meet.

Triangle.—A figure bounded by three lines is called a triangle.

A triangle of which one of the angles is a right angle is called a *right-angled* triangle (Fig. 3); and the side which joins the two sides containing the right angle is called the *hypothenuse* (or hypotenuse). If all the sides of a triangle are equal, the triangle is *Equilateral.* If it has two sides equal, the triangle is *Isosceles.* If the sides are all unequal, the triangle is *Scalene.*

Polygon.—A figure having more than four sides is called a *polygon.* Polygons are of two classes, *regular* and *irregular.*

Irregular Polygons have their sides and angles unequal.

Regular Polygons have all their sides and angles equal, and possess the property (an important one for us) that they can always be inscribed in circles; in other words, a circle can always be drawn through the angular points of a regular polygon (Figs. 12 and 13).

Special names are given to regular polygons, according to the number of sides they possess; thus, a polygon of five sides is a *pentagon;* of six sides, a *hexagon;* of seven, a *heptagon;* of eight, an *octagon;* and so on.

Quadrilaterals.—All figures bounded by four lines are called quadrilaterals. The most important of these are the *square* and *oblong* or *rectangle.* In a square (Fig. 4*a*) the sides are all equal and the angles all right angles, and consequently equal. An oblong or rectangle has all its angles right angles, but only its opposite sides are equal. (Fig. 4*b*.)

Fig. 4*a*.

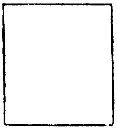


Fig. 4*b*.

Circle.—A circle is a figure bounded by a curved line such that all points in the line are at an equal distance from a certain point within the figure, which point is called the *centre.*

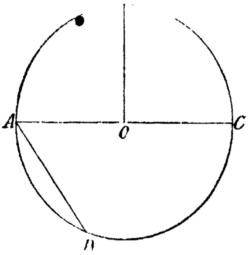
The bounding line of a circle is called its *circumference.* A part only of the circumference, no matter how large or small, is called an *arc.* An arc containing a quarter of the circumference is a *quadrant.* An arc containing half the circumference is a *semicircle.* A line drawn from the centre to any point in the circumference is a *radius* (plural, *radii*). The line joining the extremities of any arc is a *chord.* A chord that passes through the centre is a *diameter.*

A line drawn from the centre of, and perpendicular to, any chord that is not a diameter of a circle, will pass through its centre.

In practice a circle, or arc, is 'described' from a chosen, or given, centre, and with a chosen, or given, radius.

If two circles have a common centre, their circumferences are always the same distance apart.

Fig. 5.



In Fig. 5.

O is the centre.

A D (the curve) is an arc.

A B (or B C) is a quadrant.

A C B is a semicircle.

O A (or O B, or B C) is a radius.

A D (the straight line) is a chord.

A C is a diameter.

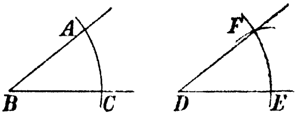
PROBLEM I.

To draw an angle equal to a given angle.

CASE I.—Where the 'given' angle is given by a drawing.

This problem, though simple, is often very useful in practice, especially for elbows, where the angle (technically called 'rake' or 'bevil') is marked on paper, and has to be copied.

FIG. 6.



Let A B C (Fig. 6) be the given angle. With B as centre and radius of any convenient length, describe an arc cutting B A, B C (which may be of any length, *see* Def.) in points A and C. Draw any line D E, and with D as centre and same radius as before, describe an arc cutting D E in E. With E as centre and the straight line distance from A to C as radius, describe an arc intersecting in F the arc just drawn. From D draw a line through F; then the angle F D E will be equal to the given angle A B C.

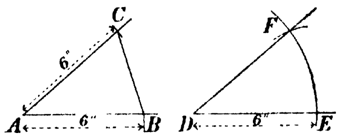
CASE II.—Where the given angle is an angle in already existing fixed work.

The angle to which an equal angle has to be drawn, may be an angle existing in already fixed work, fixed piping for instance; or in brickwork, when, suppose, a cistern may have to be made to fit in an angle between two walls. In such cases a method often used in practice is to open a two-fold rule in the angle which is to be copied. The rule is then laid down on the working surface, whatever it may be (paper, board, &c.), on which the work of drawing an angle equal to the existing angle has to be carried out, and lines are drawn on that surface, along either the outer or inner edges of the rule. The rule being then removed, the lines are produced; meeting, they give the angle required.

Case III.—Where the given angle is that of fixed work, and the method of Case II. is inapplicable.

With existing fixed work, the method of Case II. is not always practicable. A corner may be so filled that a rule cannot be applied. The method to be now employed is as follows. Draw lines on the fixed work, say piping, each way from the angle; and on each line, from the angle, set off any the same distance, say 6 in., and measure the distance between the free ends of the 6-in. lengths. That is, if A C, A B (Fig. 7) represent the lines drawn on the piping, measure the distance between B and C. Now on the working surface on which the drawing is to be made, draw any line D E, 6 in. long; and with D as centre and radius D E, describe an arc. Next, with E as centre, and the distance just measured between B and C as radius, describe an arc cutting the former arc in F. Join F D; then the angle F D E will be equal to the angle of the piping.

Fig. 7.



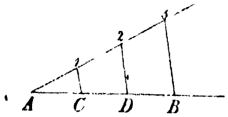
Note.—When points are 'joined,' it is always by *straight* lines.

PROBLEM II.

To divide a line into any number of equal parts.

Let A B (Fig. 8) be the given line. From one of its extremities, say A, draw a line A 3 at any angle to A B, and on it, from the angular point, mark off as many parts,—of any convenient length, but all equal to each other,—as A B is to be divided into. Say that A B is to be divided into three equal parts, and that the equal lengths marked off on A 3 are A to

Fig. 8.



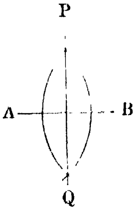
1, 1 to 2, and 2 to 3. Then join point 3 to the B extremity of A B, and through the other points of division, here 1 and 2, draw lines parallel to 3 B, cutting A B in C and D. Then A B is divided as required.

PROBLEM III.

To bisect (divide a line into two equal parts) a given line.

Let A B (Fig. 9) be the given line. With A as centre, and any radius greater than half its length, describe an indefinite arc; and with B as centre and same radius, describe an arc intersecting the former arc in points P and Q. Draw a line through P and Q; this will bisect A B.

Fig. 9.



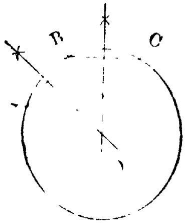
Note.—It is quite as easy to bisect A B by Problem II.; but the method shown gives, in P Q, not only a line bisecting A B, but a line perpendicular to A B. This must be particularly remembered.

PROBLEM IV.

To find the centre of a given circle.

Let A B C (Fig. 10) be the given circle. Take any three points A, B, C, in its circumference. Join A B, B C; then A B, B C, are chords (*see* Def.) of the circle A B C. Bisect A B, B C; the point of intersection, O, of the bisecting lines is the centre required.

FIG. 10.



PROBLEM V.

To describe a circle which shall pass through any three given points that are not in the same straight line.

Let A, B, C (Fig. 10) be the three given points. Join A B, B C. Now the circle to be described will not be a circle through A, B, C, unless A B, B C, are chords of it. Let us therefore assume them such, and so treating them, find (by Problem IV.) O the centre of that circle. With O as centre, and the distance from O to A as radius, describe a circle; it will pass also through B and C, as required.

PROBLEM VI.

Given an arc of a circle, to complete the circle of which it is a portion.

Let A C (Fig. 10) be the given arc; take any three points in it as A, B, C; join A B, B C Bisect A B, B C by lines intersecting in O. With O as centre, and O to A or to any point in the arc, as radius the circle can be completed.

PROBLEM VII.

To find whether a given curve is an arc of a circle.

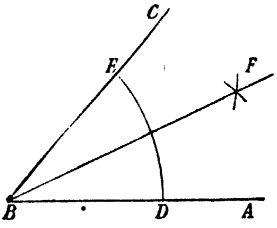
Choose any three points on the given curve, and by Problem V describe a circle passing through them. If the circle coincides with the given curve, the curve is an arc.

PROBLEM VIII.

To bisect a given angle.

Let A B C (Fig. 11) be the given angle. With B as centre and any convenient radius describe an arc cutting A B, B C in D and E. With D and E as centres and any convenient distance, greater than half the length of the arc D E as radius describe arcs intersecting in F. Join F to B; then F B bisects the given angle.

FIG. 11.

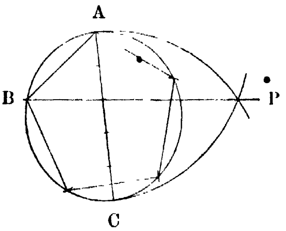


PROBLEM IX.

In a given circle, to inscribe a regular polygon of any given number of sides

Divide (PROBLEM II.) the diameter A C of the given circle (Fig. 12) into as many equal parts as the figure is to have sides, here say five. With A and C as centres, and C A as radius, describe arcs intersecting in P. Through P and the second point of division of the diameter draw a line P B cutting the circumference in B; join B A, then B A will be one side of the required figure. Mark off the length B A from A round the circumference until a marking off reaches B. Then, beginning at point A, join each point in the circumference to the next following; this will complete the polygon.

FIG. 12.



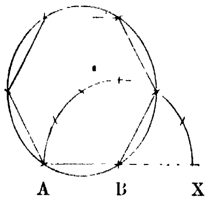
NOTE.—By this problem a circumference, and therefore also one-half of it (semicircle), one-third of it, one-fourth of it (quadrant), and so on, can be divided into any number of equal parts.

PROBLEM X.

To describe any regular polygon, the length of one side being given.

Let A B (Fig. 13) be the given side of, say, a hexagon. With either end, here B, as centre and the length of the given side as radius, describe an arc. Produce A B to cut the arc in X. Divide the semicircle thus formed into as many equal parts (PROBLEM IX., NOTE) as the figure is to have sides (six), and join B to the second division point of the semicircle counting from X. This line will be another side of the required polygon. Having now three points, A, B, and the second division point from X, draw a circle through them (Problem V.), and, as a regular polygon can always be inscribed in a circle (*see* Def.), mark off the length B A round

Fig. 13.



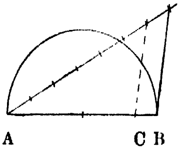
the circumference from A until at the last marking-off, the free extremity of the second side (the side found) of the polygon is reached, then, beginning at A, join each point in the circumference to the next following; this will complete the polygon (hexagon).

PROBLEM XI.

To find the length of the circumference of a circle, the diameter being given.

Divide the given diameter A B (Fig. 14) into seven equal parts (Problem II.). Then three times A B, with C B, one of the seven parts of A B, added, that is with one-seventh of

Fig. 14



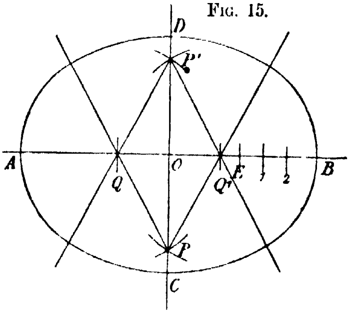
A B added, will be the required length of the circumference. The semicircle of the figure is superfluous, but may help to make the problem more clearly understood.

PROBLEM XII.

To draw an oval, its length and width being given.

Draw two lines A B, C D (the axes of the oval), perpendicular to one another (Fig. 15), and intersecting in O. Make O A and O B each equal to half the length, and O C and O D each equal to half the width of the oval. From A mark off A E equal to C D the width of the oval, and divide E B into three equal parts. With O as centre and radius equal to two of the parts, as E 2, describe arcs cutting A B in points Q and Q′. With Q and Q′ as centres and Q Q′ as radius describe arcs intersecting C D in points P and P′. Join P Q, P Q′, P′ Q and P′ Q′; in these lines produced the end and side curves must meet. With Q and Q′ as centres and Q A as radius, describe the end curves, and with P and P′ as centres and radius P D, describe the side curves; this will complete the oval.

FIG. 15.



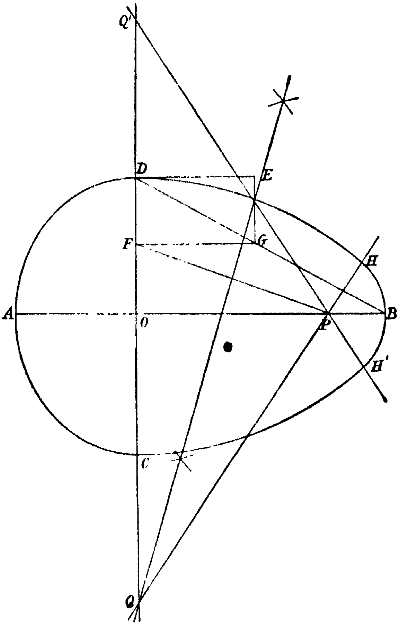
NOTE.—Unless care is taken, it may be found that the end and side curves will not meet accurately, and even with care this may sometimes occur. It is best if great accuracy be required in the length, to draw the end curves first, and then draw side curves to meet them; or, if the width is most important, to draw the side curves first. The centres (P and P′) for the side curves come inside or outside the curves, according as the oval is broad or narrow. This figure is sometimes erroneously called an ellipse. It is, however, a good approximation to one, and for most purposes where an elliptical article has to be made, is very convenient.

PROBLEM XIII.

To draw an egg-shaped oval, having the length and width given.

Make A B (Fig. 16) equal to the length of the oval, and from A set off A O equal to half its width. Through O draw an indefinite line Q Q′ perpendicular to A B, and with O as centre and O A as radius describe the semicircle C A D. Join D B; and from D draw D E perpendicular to Q Q′ and equal to O D. Also from E draw E G parallel to Q Q′ and

FIG. 16.



intersecting D B in G, and from G draw G F parallel to D E and intersecting Q Q′ in F. From B set off B P equal to D F, and join P F. Bisect F P and through the point of bisection draw a line cutting Q Q′ in Q. Join Q P and produce it indefinitely, and with Q as centre and Q D as radius describe an arc meeting Q P produced in H. Make O Q′ equal to O Q, and join Q′ P and produce it indefinitely. With Q′ as centre, and Q′ C (equal to Q D) as radius, describe an arc meeting Q′ P produced in H′. And with P as centre and P B as radius describe an arc to meet the arcs D H and C H′ in H and H′; and to complete the egg-shaped oval.

PROBLEM XIV.

To describe an ellipse.

Before working this as a problem in geometry, let us draw an ellipse non-geometrically and get at some sort of a definition. This done, we will solve the problem geometrically, and follow that with a second mechanical method of describing the curve.

METHOD I.—MECHANICAL.

A. *Irrespective of dimensions.*—On a piece of cardboard or smooth-faced wood, mark off any two points F, F′ (Fig. 17), and fix pins securely in those points. Then take a piece of thin string or silk, and tie the ends together so as to form a loop; of such size as will pass quite easily over the pins. Next, place the point of a pencil in the string, and take up the slack so that the string, pushed close against the wood, shall form a triangle, as say, F D F′, the pencil point being at D. Then, keeping the pencil upright, and always in the string, and the string taut, move the pencil along from left to right say, so that it shall make a continuous mark. Let us trace the course of the mark. Starting from D, the pencil, constrained always by the string, moves from D to P, then on to B, P′, C, P^2, P^3, A, P^4, and D again, describing a curve which returns into itself; this curve is an ellipse.

Having drawn the ellipse let us remove the string and pins, draw a line from F to F′, and produce it both ways to terminate in the curve, as at B and A. Then A B is the *major axis* of the ellipse, and F, F′ are its *foci*. The mid-point of A B is the *centre* of the ellipse. Any line through the centre and terminating both ways in the ellipse is a *diameter*. The major axis is the longest diameter, and is commonly called the length of the ellipse. The diameter through the centre at right angles to the major axis is the shortest diameter, or *minor axis*, or width of the ellipse.

Referring to the Fig.:—

A D P B C is an ellipse.

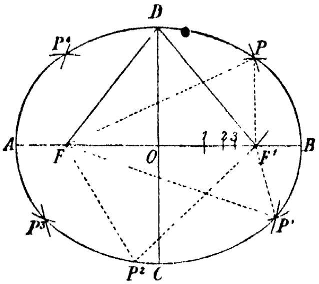
F, F′ are its foci (singular, *focus*).

A B is the major axis.

C D is the minor axis.

O is the centre.

FIG. 17.



We notice with the string and pencil that when the pencil point reaches P, the triangle formed by the string is F P F′; when it reaches P′, the triangle is F P′ F′; when it reaches P^2 the triangle is F P^2 F′; and when P^3 is reached, it is F P^3 F′. Looking at these triangles, it is obvious that

F F′ is one side of each of them; from which it follows, seeing that the loop of string is always of one length, that the sum of the other two sides of any of the triangles is equal to the sum of the other two sides of any other of them; that is to say, F D added to D F′ is equal to F P added to P F′, is equal to F P′ added to P′ F′, and so on.

Which leads us to the following definition.

DEFINITION.

Ellipse.—The ellipse is a closed curve (that is, a curve returning into itself), such that the sum of the distances of any point in the curve from certain two points (foci), inside the curve is always the same.

B. *Length and width given.*—Knowing now what an ellipse is, we can work to dimensions. Those usually given are the length (major axis), and width (minor axis). Draw A B, C D (Fig. 17), the given axes, and with either extremity, C or D, of the minor axis as centre, and half A B, the major axis as radius, describe an arc cutting A B in F and F′. Fix pins securely in F, F′ and D (or C). Then, having tied a piece of thin string or silk firmly round the three pins, remove the pin at D (or C); put, in place of it, a pencil point in the string; and proceed to mark out the ellipse as above explained.

METHOD II.—GEOMETRICAL.—THE SOLUTION OF THE PROBLEM. LENGTH AND WIDTH GIVEN.

Draw A B, C D (Fig. 17), the major and minor axes. With C or D as centre, and half the major axis, O B say, as radius, describe arcs cutting A B in F and F′. On A B, and between O and F′, mark points—any number and anywhere, except that it is advisable to mark the points closer to each other as they approach F′. Let the points here be 1, 2, and 3. With F and F′ as centres and A 2, B 2 as radii respectively, describe arcs intersecting in P; with same centres and A 3, B 3 as radii respectively, describe arcs intersecting in P′. With F′ and F as centres and A 3, B 3 as radii respectively, describe arcs intersecting in P^3. With same centres and A 2, B 2 as radii respectively, describe arcs intersecting in P^4. Similarly obtain P^2. We have thus nine points, D, P, B, P', C, P^2, P^3, A and P^4, through which an even curve may be drawn which will be the ellipse required. A greater number of points through which to draw the ellipse may of course be obtained by taking more points between O and F', and proceeding as explained.

METHOD III.—Mechanical.—Length and Width given.

As it is not always possible to proceed as described at end of *Method I.*, for pins cannot always be fixed in the material to be drawn upon, we now give a second mechanical method. Having drawn (Fig. 18*b*) A B, C D, the given axes, then, on a strip of card or stiff paper X X (Fig. 18*a*), mark off from one end P, a distance P F equal to half the major axis (length), and a distance P E equal to half the minor axis (width). Place the strip on the axes in such a position that the point E is on the major axis, and the

Fig. 18*a*.

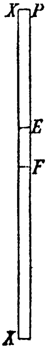
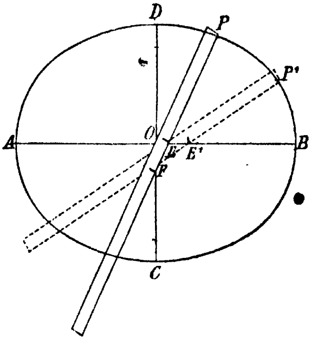


Fig. 18*b*.



point F on the minor, and mark a point against the point P. Now shift X X to a position in which E is closer to B, and F closer to C, and again mark a point against P. Proceed similarly to mark other points, and finally draw an even curve through all the points that have been obtained.

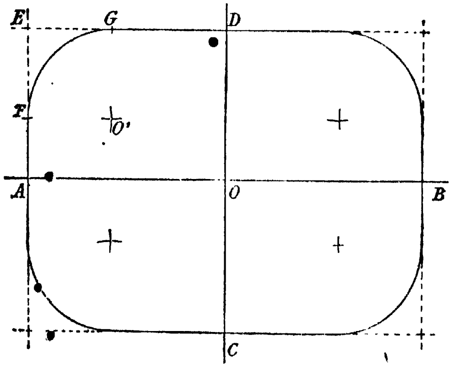
The following problems deal with shapes often required by the metal-plate worker, and will give him an idea of how to adapt to his requirements the problems that precede. The explanation of the measurement of angles that concludes the chapter will further assist him in his work.

PROBLEM XV.

To draw an oblong with round corners.

Draw two indefinite lines A B, C D (Fig. 19) perpendicular to one another and intersecting in O. Make O A and O B each equal to half the given length; and O C and O D each equal to half the given width. Through C and D

Fig. 19.



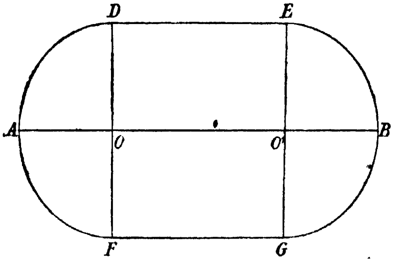
draw lines parallel to A B, and through A and B draw lines parallel to C D. We now have a rectangle or oblong, and require to round the corners, which are quadrants. Mark off from E along E D and E A equal distances E G and E F according to the size of corner required. With F and G as centres and E F or E G as radius, describe arcs intersecting in O′ With O′ as centre and same radius describe the corner F G. The remaining corners can be drawn in similar manner.

PROBLEM XVI.

To draw a figure having straight sides and semicircular ends (oblong with semicircular ends).

Draw a line A B (Fig. 20) equal to the given length; make A O and B O′ each equal to half the given width.

Fig. 20.



Through O and O′ draw indefinite lines perpendicular to A B; with O and O′ as centres and O A as radius describe arcs cutting the perpendiculars through O and O′ in D F and G E. Join D E, G F; this will complete the figure required.

ANGLES AND THEIR MEASUREMENT.

The right angle B O C (Fig. 5) *subtends* the quadrant B C. If we divide that quadrant into 90 parts and call the parts *degrees*, then a right angle subtends or contains 90 degrees (written 90°), or as usually expressed, *is an angle of* 90 degrees, the degree being the unit of measurement. If each division point of the quadrant is joined to O, the right angle is divided into 90 angles, each of which subtends or is an angle of 1 degree. That is to say, an angle is measured by the number of degrees that it contains Suppose the quadrant B A is divided as was B C, then B O A also is an angle of 90 degrees. If the division is continued round the semicircle A D C, this will contain 180 degrees, and the whole circumference has been divided into 360 degrees. As an angle of 90, which is a fourth part of 360 degrees, subtends a quadrant or fourth part of the circumference of the circle, so an angle of 60, which is a sixth part of 360 degrees, subtends a sixth part of the circumference, and similarly an angle of 30 degrees subtends a twelfth part, an angle of 45 an eighth part, and so on. And this angular measurement is quite independent of the dimensions of the circle; the quadrant always subtends a right angle; the 60 degrees angle always subtends an arc of one sixth of the circumference; and the like with other angles From our definition p. 5 we have it that a chord is the line joining the extremities of any arc. The chord of a sixth part of the circumference of any circle, we have now to add, is equal to the radius of that circle. This being the case, and as an angle of 60 degrees subtends the sixth part of the circumference of a circle, it follows that an angle of 60° subtends a chord equal to the radius.

SCALE OF CHORDS.

Construction.—We have now the knowledge requisite for setting out a scale of chords, by which angles may be drawn and measured.

On any line O B (Fig. 21) describe a semicircle O A B, and from its centre C draw C A perpendicular to O B. Divide O A into nine equal parts. Then, as O A, being a quadrant, contains 90°, each of the nine divisions will contain 10°. The points of division, from O, of the quadrant, are marked 10, 20, 30, &c., up to 90 at A. With O as centre, describe arcs from each of these division points, cutting the line O B. Note that the arc from point 60 cuts O B in C, the centre of the semicircle; the chord from O to 60 (not drawn in the Fig.), that is, the chord of one-sixth of the circumference of the circle whose centre is C, being equal to the radius of that circle. Draw a line O E parallel to O B, and from O let fall O O perpendicular to O B. Also from each of the points where the arcs cut O B let fall perpendiculars to O B and number these consecutively to correspond with the numbers on the quadrant O A. The scale is now complete.

FIG. 21.

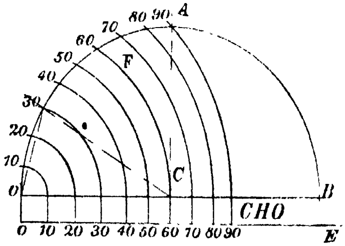
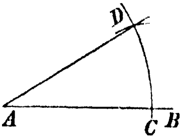


FIG. 22.



How to use. It is used in this way. Suppose from a point A in any line A B (Fig 22) we have to draw a line at an angle of 30° with it. Then with A as centre and the distance from O to 60 on O E (Fig. 21) as radius, describe an arc C D cutting A B in C. And with C as centre and the distance from O to 30 on O E (Fig. 21) (the angle to be drawn is to be of 30°) as radius describe an arc intersecting arc C D in D. Join D A, then D A C will be the required angle of 30°. Similarly with angles of other dimensions.

In taking the lengths of arcs, we really take the length of their chords, and it is these lengths that (Fig. 21) we have set off along O E. The angle (Fig. 21) O C F (the point F is the point 60) being an angle of 60° subtends a chord equal to the radius; therefore in O to 60 we have the radius C O. In the example (Fig. 22), the distance C D (O to 30) is the chord of 30°; and it is clear that we must set this off on an arc C D of a circle of the same size as that employed in the construction of the scale, and this we do by making A C equal O to 60 on the same scale.

When a scale of chords has been constructed as explained, the semicircle may be cut away, and we thus get a scale convenient for shop use in the form of a rule.

CHAPTER III.

PATTERNS FOR ARTICLES OF EQUAL TAPER OR INCLINATION.

(CLASS I.)

(4.) It is necessary here at once to remark that ordinary workshop parlance speaks of 'slant,'—not as meaning an *angle*, but a *length*; not as referring to the angle of inclination of a tapering body, but to the length of its slanting portion. It is in this sense that we shall use the word, and shall employ the word 'taper' or the term 'inclination of slant' when meaning an angle.

(5.) In order that the rules for the setting out of patterns for articles of equal taper or inclination may be better understood and remembered, it is advisable to consider the principles on which the rules are based, as a knowledge of principles will often enable a workman himself to find rules for the setting out of patterns for odd work. The basis of the whole of the articles in this Class is the right cone. It is necessary, therefore, to define the right cone and explain some of its properties.

DEFINITION.

(6.) *Right Cone.*—A right cone is a solid figure generated or formed by the revolution of a right-angled triangle about one of the sides containing the right angle. The side about which the triangle revolves is the *axis* of the cone; the other side containing the right angle being its *radius*. The point of the cone is its *apex*; the circular end its *base*. The hypotenuse of the triangle is the slant of the cone. From the method of formation of the right cone, it follows that the axis is perpendicular to the base. The height of the cone is the length of its axis

(7.) Referring to Fig. 1*a*, O B E represents a cone generated or formed by the revolution of the right-angled triangle O A B (Fig. 1*b*) about one of its sides containing the right angle, here the side O A. Similarly the cone O D F,

FIG. 1*a*.

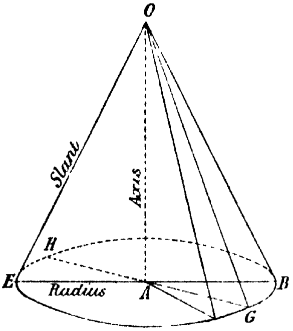


FIG. 1*b*.

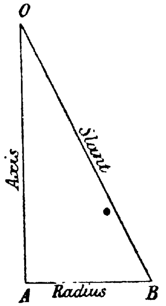


Fig. 2*a*, is formed by the revolution of O C D (Fig. 2*b*) about its side O C. As will be seen from the figs., O A, O C are respectively the axes of the cones O B E, O D F, as also their heights. Their bases are respectively B G E H, D K F L,

FIG. 2*a*.

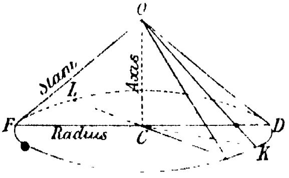
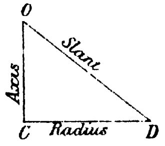


FIG. 2*b*.



and the radii of the bases are A B and C D. The slants of the cones are O B and O D, the apex in either being the point O. Other lines will be seen in figs., namely, those representing the revolving triangle in its motion of generating the cone. The sides of these triangles that start from the apex and terminate in the base are all equal, it must be borne in mind; and each of them is the slant of the cone. Likewise their sides that terminate in A are all equal, and each shows a radius of the base of the cone. How these particulars of the relations to one another of the several parts of the *right cone* apply in the setting-out of patterns will be seen in the problems that follow.

PROBLEM I.

To find the height of a cone, the slant and diameter of the base being given.

Draw any two lines O A, B A (Figs. 3 and 4) perpendicular to each other and intersecting in A. On either line mark off from A half the diameter of the base, in other words, the radius of the base, as A B. With B as centre, and radius equal to the slant, describe an arc cutting A O in O. Then O A is the height of the cone.

Fig. 3.

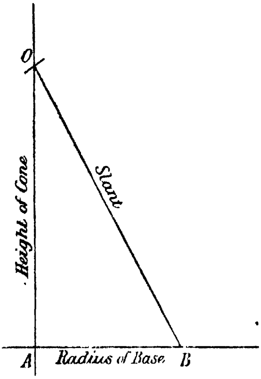
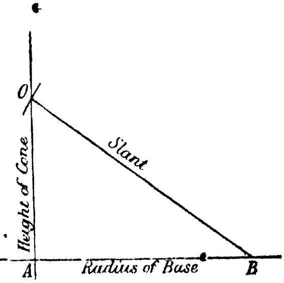


Fig. 4.



PROBLEM II.

To find the slant of a cone, the height and diameter of the base being given.

Draw any two lines O A, B A (Figs. 3 and 4) perpendicular to each other and intersecting in A. On either line mark off from A half the diameter of the base (radius of the base), as A B, and make A O on the other line equal to the height of the cone; join O B. Then O B is the required slant

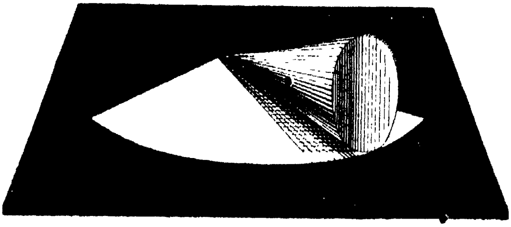
CHAPTER IV.

PATTERNS FOR ROUND ARTICLES OF EQUAL TAPER OR INCLINATION OF SLANT.

(CLASS I. *Subdivision a.*)

(8.) If a cone has its inclined or slanting surface painted say, white, and be rolled while wet on a plane so that every portion of the surface in succession touches the plane, then the figure formed on the plane by the wet paint (see Fig. 5)

FIG. 5



will be the pattern for the cone. As the cone rolls (the figure represents the cone as rolling), the portion of it touching the plane at any instant is a slant of the cone (see § 7).

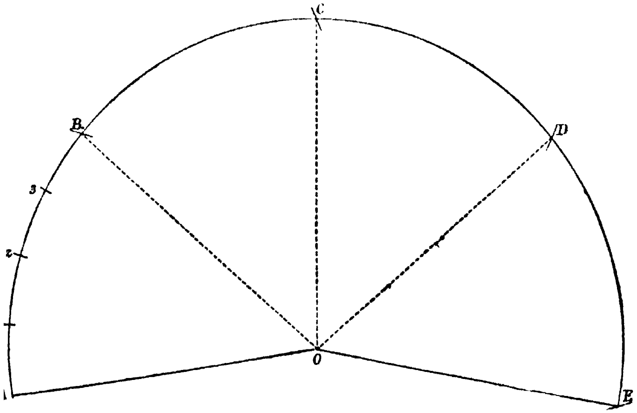
(9.) Examining the figure formed by the wet paint, we find it to be a *sector* of a circle, that is, the figure contained between two radii of a circle and the arc they cut off. The length of the arc here is clearly equal to the length of the circumference of the base of the cone, and the radius of the are evidently equal to the slant of the cone. From this it is obvious that to draw the pattern for a cone, we require to know the slant of the cone (which will be the radius for the pattern), and the circumference of the base of the cone.

PROBLEM III.

To draw the pattern for a cone, in one piece or in several pieces, the slant and diameter of the base being given.

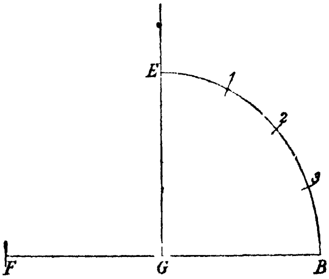
Pattern in one Piece.—With O A (Fig. 6*b*) equal to the slant as radius, describe a long arc A C E. What has now to be done is to mark off a length of this arc equal to the circumference of the base of the cone. The best and quickest way for this is as follows. Draw a line F B (Fig. 6*a*) equal to the given diameter of the base, and bisect it in G; then G B is a radius of the base. From G draw G E perpendicular

Fig. 6*b*.



to F B; and with G as centre and radius G B describe from B an arc meeting G E in E. The arc B E is a quadrant (quarter) of the circumference of the base of the cone. Divide this quadrant into a number of equal parts, not too many, say four, by points 1, 2, 3. From A (Fig. 6*b*) mark off along arc A C E four parts, each equal to one of the divisions of the quadrant, as from A to B. Take this length A B equal to the four parts, that is, equal to the quadrant, and from B set it off three times along the arc towards E as from B to C, C to D, D to E. Join E to O; then O A C E O will be the pattern required.

FIG. 6*a*.



NOTE.—It must be noted that when this pattern is bent round to form the cone, the edges O A and O E will simply butt up against each other, for no allowance has been made for *lap* or *seam*. Let us call the junction of O A and O E the line of butting. Nor, further, has any allowance been made for *wiring* of the edge A C E. These most essential matters will be referred to immediately.

PATTERN IN MORE THAN ONE PIECE.—If B be joined to O, then the sector O A B will be the pattern for *one-quarter* of the cone. If C be joined to O, then the sector O A C is the pattern for *one-half* of it. Similarly O A D will give *three-quarters* of the cone. A cone pattern can thus be made in one, two, three, or four pieces. If the cone is required to be made in three pieces, then instead of dividing, as above, a quadrant of the circumference of the base, divide one-third of it into parts, say five; set off five of the parts along A C E from A, and join the last division point to the centre; the sector so obtained will be the pattern for *one-third* of the cone. If required to be made in five pieces, divide a fifth of the circumference of the base into equal parts, and proceed as before. Similarly for any number of pieces that the pattern may be required in.

PROBLEM IV.

To draw the pattern for a cone, the height and the diameter of the base being given.

First find the slant O B (Fig. 7*a*) by PROBLEM II. Then with A as centre and radius A B, describe B C a quadrant of the circumference of the base, and proceed, as in PROBLEM III., to draw the pattern Fig. 7*b* (the plain lines).

FIG. 7*a*.

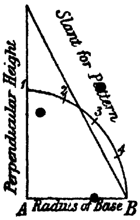
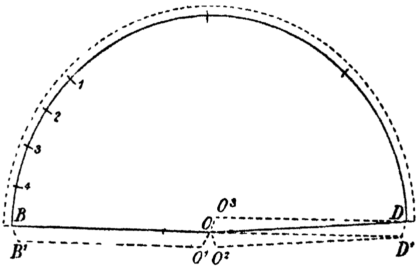


FIG. 7*b*.



ALLOWANCE FOR LAP, SEAM, WIRING, &c.

(10.) It has already been stated that the geometric pattern Fig. 6*b* has no allowance for seam, wiring, or edging. (For the present it is assumed that these terms are understood; we shall come back to them later on.) In the pattern Fig. 7*b* the dotted line O′ B′ parallel to the edge O B shows 'lap' for soldered seam. For a 'grooved' seam not only must there be this allowance, but there must be a similar allowance along the edge O D. These allowances, it must be distinctly remembered, are always *extras* to the geometric pattern; that is to say, the junction line of O D and O B, or line of butting (*see* NOTE, Problem III) is not interfered with. And here a word of warning is necessary. Suppose instead of marking off a parallel slip or lap for soldered seam, a slip D O D′ going off to nothing at the centre O, is marked off, and that then, for soldering up, there is actually used not this triangular slip, but a parallel one as D D′ O O^3, the result brought about will be that the work will solder up untrue; there will be, in fact, a 'rise' at the base of the work. We can understand the result in this way. If the parallel slip D D′ O O^3 used for soldering were cut off, there would remain a pattern which is not the geometric pattern, but a nondescript approximation, having a line of butting other than the true line. And it being thus to an untrue pattern that the parallel slip for seam is added, the article made up from the untrue pattern must of course itself necessarily be untrue. In the fig. the dotted line parallel to the curve of the pattern shows an allowance for wiring. For a grooved seam there must be on the edge O D an addition O D D′ O^2 similar to the addition on the edge O B, as above stated.

(11.) In working from shop patterns for funnels, oil-bottle tops, and similar articles, workmen often find that if they take a good lap at the bottom, and almost nothing at the top of the seam, the pattern is true. And so it is, for these patterns have the triangular slip D O D′ added. Whereas, if a parallel piece D O^3 O D′ is used for lap, the pattern is untrue. Which again is the case, because, now, in addition to D O D′, an extra triangular piece D O^3 O is used, and this extra *is taken off the geometric pattern.* Consequently,* the line of butting is interfered with; that is to say, the two lines O B and O D, instead of meeting, overlap; O B forming a junction with O^3 D instead of with O D; with which O B must always form a junction, for the pattern to be true. In etting out patterns, to prevent error, the best rule to follow nd adopt is, to first mark them out independent of any llowance for seams, or wiring, or edging, and to afterwards dd on whatever allowances are intended or requisite. In ture diagrams, allowances, where shown, will be mostly hown by dotted lines.

EFINITION.

(12.) FRUSTUM.—If a right cone is cut by a plane parallel to

FIG. 8a.

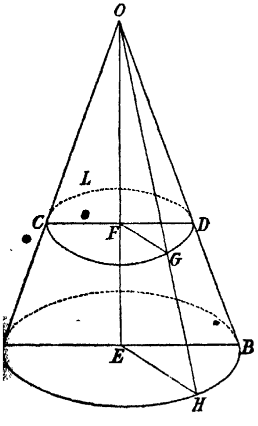
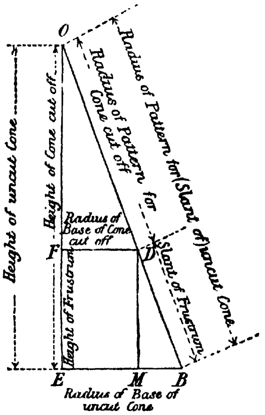


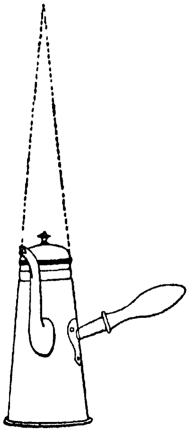
FIG. 8b.

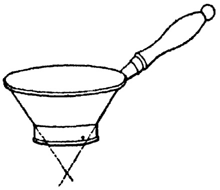

O
Radius of Pattern for (Slant of) uncut Cone
Radius of Pattern for Cone cut off
Height of uncut Cone
Height of Cone cut off
Radius of Base of Cone cut off
F
D
Slant of Frustum
Height of Frustrum
E
M
B
Radius of Base of uncut Cone

base, the part containing the apex is a complete cone, as G D L (Fig. 8a), and the part C A B D containing the base A H B K is a *frustum* of the cone. In other words a *frustum* of a right cone is a solid having circular ends, and of equal taper or inclination of slant everywhere between the ends. Conversely a round equally tapering body having top and base parallel is a frustum of a right cone.

(13.) Comparing such a solid with round articles of equal taper or inclination of slant, as pails, coffee-pots, gravy strainers, and so on (Fig. 9), it will be seen that they are portions (frusta) of right cones.

FIGS. 9.

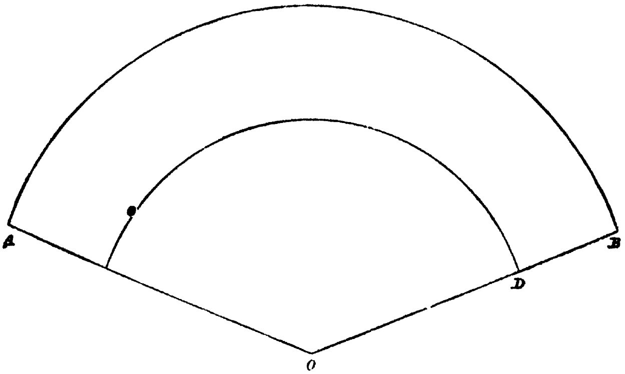




(14.) In speaking here of metal-plate articles as portions of cones, it must be remembered that all our patterns are of *surfaces*, seeing that we are dealing with metals in *sheet*; and that these patterns when formed up are not solids, but merely simulate solids. It is, however, a convenience, and leads to no confusion to entirely disregard the distinction; the method of expression referred to is therefore adopted throughout these pages.

(15.) By Fig. 8*b* is shown the relations of the cone O A B of Fig. 8*a* with its portions O C D (complete cone cut off), and C A B D (frustum). The portion O C D is a complete cone, as it is the solid that would be formed by the revolution of the right-angled triangle O F D (both figs.) around O F. The triangles O F G and O F C (Fig. 8*a*) represent the triangle O F D in progress of revolution. The triangle O E B (both figs.) is the triangle of revolution of the uncut cone O A B (Fig. 8*a*) and O E H, O E A represent O E B in progress of revolution. The height of the cone O A B being O E (both figs.), the height of the cone O C D is O F (both figs.). The radius for the construction of pattern of the uncut cone O A B will be O B (both figs.) for the pattern of O C D, the cone cut off, the radius will be O D (both figs.). In F E, or D M, we have the height of the frustum. Just as (§ 8) the portion of the rolling cone touching the plane at any instant is a slant of the cone, so the slant of a frustum is that portion of it, which, if it were set rolling on a plane, would at any instant touch the plane. D B is a slant of the frustum C A B D. The extremities of a slant of a frustum are 'corresponding points.' Other details of cone and frustum are shown in Fig. 8*b*.

FIG. 10.



(16.) It is obvious that, if the patterns for the cones O A B, O C D (Fig. 8*a*) be drawn (Fig. 10) from a common centre O, the figure A C D B will be the pattern for the frustum

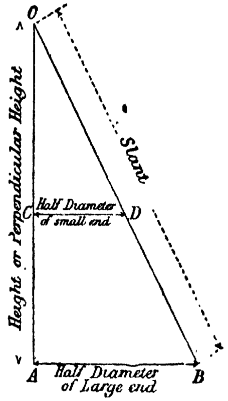
ACDB (Fig. 8*a*). From which we see that in order to draw the pattern for the frustum of a cone, we must know the slant of the cone of which the frustum is a portion, that is, we must know the radius for the construction of the pattern of that cone, and also the slant (radius for pattern) of the cone cut off.

PROBLEM V.

Given the dimensions of the ends of a round equal-tapering body (frustum of right cone), and its upright height. To find the slant, or the height, of the cone of which it is a portion.

Draw any two lines O A, A B (Fig. 11) at right angles to each other and intersecting in A. From A on either line, say on B A, mark off A B equal to half the diameter of the larger of the given ends, and from A on the other line make A C equal to the given upright height. Draw a line C D

FIG. 11.


O
Height or Perpendicular Height
Slant
C
Half Diameter of small end
D
A
Half Diameter of Large end
B

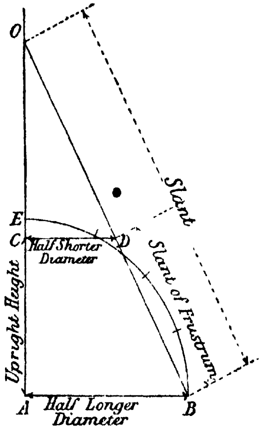
parallel to A B, or, which is the same thing, at right angles to A O, and make C D equal to half the diameter of the smaller end. Join B D, and produce it, meeting A O in O. Then O A is the height of the cone of which the tapering body is a portion, and O B the slant.

PROBLEM VI.

To draw the pattern for a frustum of a cone, the diameters of the ends of the frustum and its upright height being given.

The Frustum.—Draw any two lines O A, B A (Fig. 12*a*) perpendicular to each other and meeting in A; on one of the perpendiculars, say B A, make A B equal to half the longer diameter (radius), and on the other make A C equal to the given upright height. Draw a line C D perpendicular to A O and make C D equal to half the shorter diameter. Join B D, and produce it, meeting A O produced in O. With A

FIG. 12*a*.



as centre, and radius AB, describe quadrant B E, which divide into any convenient number of equal parts, here four.

To draw the pattern (Fig. 12b) take any point O′ as

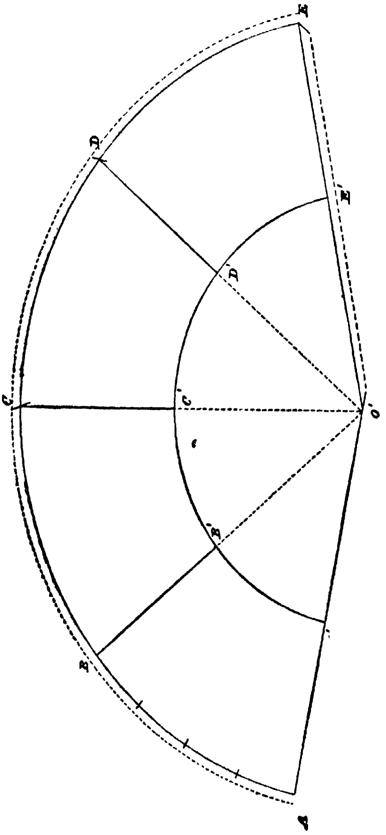


FIG. 12b

centre, and with radius O B (Fig. 12*a*) describe an arc A C E; also with same centre and radius, O D (Fig. 12*a*), describe an arc A′ C′ E′. From any point in the outside curve, as A, draw a line through O′, and cutting the inner arc in A′. From A mark off successively parts equal to those into which the quadrant B E (Fig. 12*a*) is divided, and the same number of them, four, to B. And from B, along the outer curve, set off B C, C D, D E, each equal to A B. Join E O′, cutting the inner curve in E′. Then A A′ E′ E is the pattern required.

Just as O B (Fig. 12*a*) is the slant of the cone that would be generated by the revolution of right-angled triangle O A B around O A, so D B is the slant of the frustum of which A A′ E′ E (Fig. 12*b*) is the pattern. In the pattern the slant D B appears as A A′, B B′, C C′, &c.

Parts of the Frustum.—If B be joined to O′, the figure A A′ B′ B will be one-quarter of the pattern of the frustum; and if C be joined to O′, the figure A A′ C ′C will be pattern for one-half of it, and so on. The paragraph "Pattern in more than one Piece" in PROBLEM III. should be re-read in connection with this "Parts of a Frustum."

(17.) The problem next following is important, in that, in actual practice, the slant of a round equal-tapering body is very often given instead of its height, especially in cases where the taper or inclination of the slant is great; as for instance in ceiling-shades. The only difference in the working out of the problem from that of PROBLEM VI. is that the radii required for the pattern of the body are found from other data. Let us take the problem.

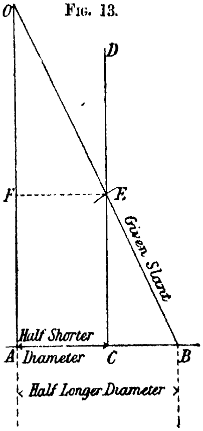
PROBLEM VII.

To draw the pattern for a round equal-tapering body (frustum of right cone), the diameter of the ends and the slant being given.

To find the required radii, draw any two lines O A, B A (Fig. 13) perpendicular to one another, and meeting in A

On either line, as A B, make A B equal to half the longer of the given diameters and A C equal to half the shorter. From C draw C D perpendicular to A B. With B as centre and

FIG. 13.



the given slant as radius, describe an arc cutting C D in E. Join B E and produce it to meet A O in O. Then O B and O E are the required radii. By E F being drawn parallel to A B, comparison may be made between this Fig. and Fig. 12*a*, and the difference between PROBLEMS VI. and VII. clearly apprehended. To draw the pattern, proceed as in PROBLEM VI.

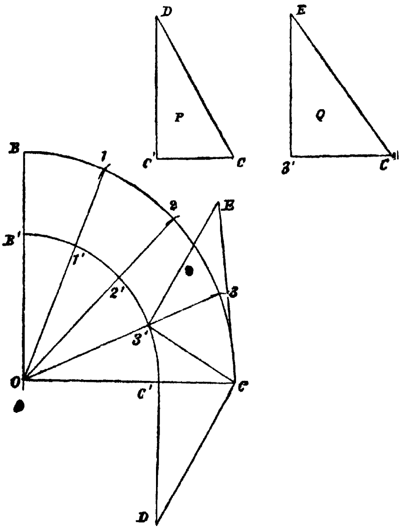
(18.) For large work and for round equal-tapering bodies which approximate to round bodies without any taper at all, the method of PROBLEM VI. is often not available, for want of space to use the long radii that are necessary for the curves of the patterns The next problem shows how to deal with such cases; by it a working-centre and long radii can be dispensed with. The method gives fairly good results.

PROBLEM VIII.

To draw, **without long radii,** *the pattern for a round equal-tapering body (frustum of right cone), the diameters of the ends and the upright height being given.*

First draw one-quarter of the plan. (To do this, we forestall for convenience what is taught in the following chapter.)

FIG. 14.

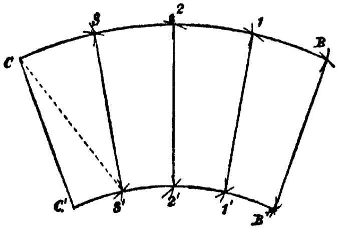


Draw any two lines B O, C O (Fig. 14) perpendicular to each other and meeting in O. With O as centre and radius equal to half the longer diameter, describe an arc meeting the lines B O, C O in B and C. With O as centre and radius equal to half the shorter diameter describe an arc B′ C′. This completes the one-quarter plan.

Now divide B C, the largest arc, into any number of equal parts, say four; and join the points of division to O by lines cutting B′ C′ in 1′, 2′, 3′. Join 3′ C, and through 3′ draw 3′ E perpendicular to 3′ C, and equal to the given upright height. Join C E; then C E may be taken as the true length of C 3′. Through C′ draw C′ D perpendicular to C O and equal to the upright height. Join C D; then C D is the true length of C C′. If it is inconvenient to find these true lengths on the plan, it may be done apart from it, as by the triangles P and Q.

To set out the pattern. Draw (Fig. 15) any line C C′ equal to C D (Fig. 14). With C′ and C as centres and radii respectively C E and C 3 (Fig. 14) describe arcs intersecting in 3 (Fig. 15). With C and C′ as centres and radii respectively C E and C′ 3′ (Fig. 14) describe arcs intersecting in 3′ (Fig. 15). Then C and 3 are two points in the outer curve of the pattern, and C′ 3′ two points in the inner curve. To find points 2 and 2′, proceed as just explained, and with the same radii, but 3′ and 3 as centres instead of C′ and C. Similarly, to find points 1′ and 1, and B′ and B. A curved line drawn from C through 3, 2, and 1 to B will be the outer curve of one-quarter of the required pattern, and a curved

FIG. 15.



line, from C′ through 3′, 2′, and 1′ to B′ its inner curve; that is C C′ B′ B is one-quarter of the pattern. Four times the quarter is of course the required pattern complete.

NOTE.—In cases where this method will be most useful, the pattern is generally required so that the article can be made in two, three, four, or more pieces. If the pattern is required in three pieces, one-third of the plan must be drawn (*see* end of Problem III., p. 31) instead of a quarter, as in Fig. 14; the remainder of the construction will then be as described above.

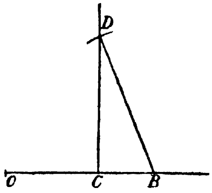
(19.) It is often desirable in the case of large work to know what the slant or height, whichever is not given, of a round equal-tapering body (frustum of right cone) will be, before starting or making the article. Here the following problems will be of service.

PROBLEM IX.

To find the slant of a round equal-tapering body (frustum of right cone), the diameters of the ends and the height being given.

Mark off (Fig. 16) from a point O in any line O B the lengths of half the shorter and longer diameters, as O C, O B.

FIG. 16.



From C draw C D perpendicular to O B. Make C D equal to the given height, and join B D. Then B D is the slant required.

PROBLEM X.

To find the height of a round equal-tapering body (frustum of right cone), the diameters of the ends and the slant being given.

Mark off (Fig. 17) from a point O in any line O B the lengths of half the shorter and longer diameters, as in PROBLEM IX., and from C draw C D perpendicular to O B. With B as centre and radius equal to the given slant, describe an arc cutting C D in E Then C E is the height required.

Essentially this problem has already been given, in the working of PROBLEM VII.

FIG. 17.

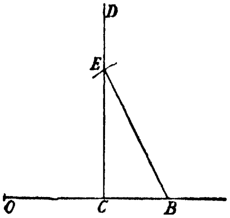
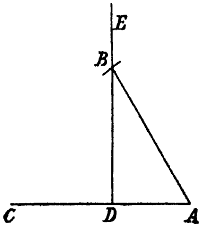


FIG. 18.



PROBLEM XI.

Given the slant and the inclination of the slant of a round equal-tapering body; to find its height.

Let A B (Fig. 18) be the slant, and the angle that A B makes with C A the inclination of the slant. From B let fall B D perpendicular to A C. Then B D is the height required.

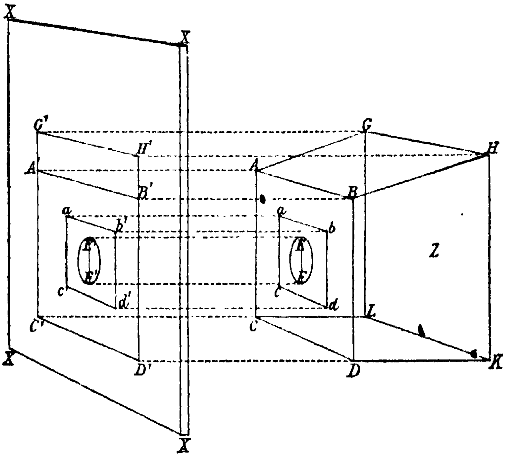
(20.) In the workshop, the inclination of the slant of a tapering body is sometimes spoken of as the body being so many inches "out of flue." This will be explained in the following chapter. If the inclination of the slant is given in these terms the problem is worked thus. From any point D in any line C A (Fig. 18) make D A equal in length to the number of inches the body is "out of flue," and draw D E perpendicular to C A. With A as centre and radius equal to the given slant, describe an arc intersecting D E in B. Then B D will be the height required.

CHAPTER V.

EQUAL-TAPERING BODIES OF WHICH TOP AND BASE ARE PARALLEL, AND THEIR PLANS.

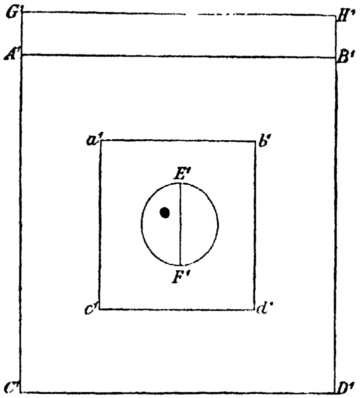
(21.) First let us understand what a plan is. Fig. 19 represents an object Z, made of tin, say, having six faces,

FIG. 19.



of which the A B C D and G H K L faces are parallel, as also the B D K H and C A L G. The A B C D and C D K L faces are square. The A B C D face has, soldered flat on it centrally, a smaller square of tin $a\,b\,c\,d$ with a central circular hole in it. Now suppose wires (represented in the fig. by dotted lines), soldered perpendicularly to the A B C D face, at A, B, C, D, *a*, *b*, *c*, *d*, E, and F (the points E and F are points at the extremities of a diameter of the circular hole). Also suppose wires soldered at G and H parallel to the other wires, and that the free ends of all the wires are cut to such length that they will, each of them, butt up against a flat surface (plane), of glass say, X X X X, parallel to the A B C D face. Lastly suppose that all the points where the wires touch the glass are joined by lines corresponding to edges of Z (see the straight lines in the figure on the plane); also that E′ and F′ are joined, that the line joining them is bisected, and a circle described passing through E′ and F′. Then the complete representation obtained is a *projection* of Z. Instead of actually *projecting* the points by wires, we may make the doing of it another supposition may, find, as if by wires, the required points, and draw the projection. The A B C D face being,

FIG. 20.



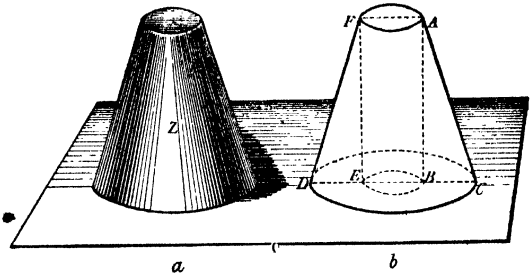
say 2 inches square, the flat piece, say 1 inch square, and the hole $\frac{1}{2}$ inch diameter, and the back face G H K L, say $2\frac{1}{4}$ in. by 2 in., then the projection that is upon the glass would be as shown in Fig. 20. The plane X X X X is here supposed vertical, and the projection G′ C′ D′ H′ is therefore an *elevation*; if the plane were horizontal, the projection would be a *plan*, and we might regard A B C D as the top of the body, and G H K L as its base, or *vice versâ*. We may define a plan then as the representation of a body obtained by projecting it on to a horizontal plane, by lines perpendicular to the plane.

(22.) The plane X X X X was supposed parallel to the A B C D face of Z; the plan A′ B′ C′ D′ of it is therefore of the same shape as A B C D, and in fact A B C D may be said to be its own plan. Similarly the G′ H′ D′ C′ is the plan of the back-face G H K L and is of the same shape as that face. But the plan of the face A G H B to which the plane is not parallel is by no means the same shape as that face, for the long edges B H and A G of the face A G H B are, in plan, the short lines B′ H′ and A′ G′. We need not, however, go farther into this, because in the case of the bodies that now concern us, the horizontal plane on which any plan is drawn is always supposed to be parallel to the principal faces of the body, so that the plans of those faces are always of the same shape as the faces. In this paragraph the plane X X X X is supposed to be horizontal.

(22*a*.) We are now in a position to explain the getting at the true length of C C′ in the fig. of PROBLEM VIII., p. 42; or, putting the matter generally, to explain the finding the true lengths of lines from their apparent lengths in their plans and elevations. Horizontal lines being excepted, there is, manifestly, for any line, however positioned in space, a vertical plane in which its elevation will appear as (if not a point) a vertical line. Let B E (Fig. 17, p. 44) be any line in the plane of the paper, and let C D be the vertical plane seen edgeways on which the elevation E C of B E is a vertical line. Then if O B be a horizontal plane seen edgeways passing through C, the line joining the B extremity of B E to the C extremity of its elevation will be the plan of B E. We get thus the figure E C B, a figure in one plane, the plane of the paper, a right-angled triangle in fact, of which the E C side is the elevation of B E, the C B side its plan, and the hypotenuse the line itself; a figure, which, as combining a line, its plan, and its elevation, we have under no other conditions than when the elevation in question is a vertical line. In the plane passing through E C and B E, that is, in the plane in which these lines wholly lie, we have in the line that we get by joining C with B the plan, full length, of B E. In respect of this plan of B E, we are concerned with no other measurement, because, in a right-angled triangle representing a line and its plan and elevation, no other measurement of the plan line can come in. Not so, however, with the elevation line of B E. Here other measurement of it than its length can and does come in, because that length varies according to the position of the vertical plane with regard to it; the plan length is always the same. But to have in the three sides of E C B, the representation of B E, and its plan and elevation, it is evident that the plane which contains B E and its plan C B must also wholly contain the elevation E C; that is, the plane must be perpendicular to the plane of the triangle. Now, no matter on what vertical plane the line B E is projected, although the length of the projection will vary, the vertical distance between its extremities, that is, its height, never varies. Hence, if, in any right-angled triangle, we have in the hypotenuse the representation of a line, in one of its sides the plan of the line, and in the other side, not necessarily the elevation that comes out vertical, but the *height* of *any* elevation of the line, it comes to the same thing as if in the latter side we had the actual elevation that is vertical. And hence, further, if we have given the plan-length of an unknown line, and the vertical distance between its extremities, we can, by drawing a line, say C B, equal to the given plan-length, then drawing from one of its extremities and at right angles to it, a line, say C E, equal to the given vertical distance, and finally joining the free extremities, as by B E, of these two lines, construct a right-angled triangle, the hypotenuse, B E, of which must be the true length of the unknown line; for there is no other line than B E of which C B and C E can be, at one and the same time, plan and elevation. We have explained this true-length matter fully, because we have to make use of it abundantly in problems to come.

(23.) Proceeding to the bodies we have to consider, we

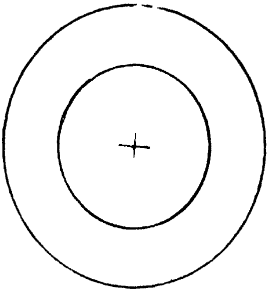
FIG. 21.



take first a frustum of a cone, Fig. 21*a*. To draw its plan, let us suppose the extremities of a diameter of its smaller face top (namely points A and F of the skeleton drawing Fig. 21*b*) (neither drawing is to dimensions), to be projected, in the way just explained, on to a plane parallel to the face, then, also as there explained, we can draw the circle which is a projection of that face. Suppose the smaller circle of Fig. 22 to be that circle, and to be to dimensions. Projecting now, similarly, the extremities of a diameter of the larger face (base), namely the points C and D of the skeleton drawing, on to the same plane, we can get the projection of the larger face. Let the larger circle of Fig. 22 be that projection. The two circular projections will be *concentric* (having the same centre) because the body Z is of equal taper, and they will, together, be the plan of Z, that is Fig. 22 is that plan.

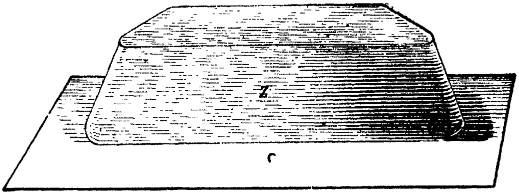
A C and F D each show the slant, and B A and E F the height. B C and D E each show the distances between the plans of corresponding points.

Fig. 22.



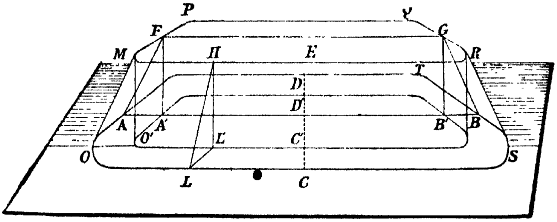
(24.) Turn to the skeleton drawing of Z. Here A C shows a slant of the frustum (§ 15), A B its height (see D M, Fig. 8*b*), and A and C are 'corresponding points' (§ 15). Looking at C D E B as at the plan of the frustum, we have, in the point B, the plan of the point A. Joining B C, we get a right-angled triangle A B C; the slant A C is its hypotenuse, the height A B is one of the sides containing the right angle, and the other side containing the right angle, B C, is the distance between the plans of the corresponding points A and C, as also between plans of corresponding points of Z anywhere. This distance is that of how much the body is 'out of flue' (a workshop expression that was referred to at the end of the previous chapter), in other words, how much A C is out of parallel with A B. What points, in the plan of a frustum, are the plans of corresponding points is shown by the fig., as the line joining the plans of corresponding points (the line joining B and C or that joining D and E, for instance) will always, if produced, pass through the centre of the circles that constitute the plan of the frustum; the centre of the circles being the plan of the apex of the cone of which the frustum is a part. Which leads us to this; that the distance, actually, between the plans of corresponding points in the plan of a frustum is equal to half the difference of the diameters of its two circles; for, the difference between E B and D C is the sum of D E and B C, and D E and B C are equal; in other words, either D E or B C is half the difference between E B and D C.

FIG. 23*a*.



(25.) Let us now consider another equal-tapering body which has top and base parallel, and we will suppose it to have flat parallel sides, flat ends, and round (quadrant) corners. Such a body is represented, except as to dimensions, in Z, Figs. 23*a* and *b*; Fig. 23*b* being a skeleton drawing of the body represented in Fig. 23*a*. Extending our definition of 'slant' to apply to such a body, a 'slant' becomes the shortest line that can be drawn anywhere on the slanting surface; and 'corresponding points' become, in accordance, the extreme points of such line. Either of the lines F A, G B, E C, H L, or M O represent a slant of the body, and F and A are corresponding points; as also are G and B, E and C, H and L, and M and O. The height of the body is represented by either of the lines F A′, G B′, E C′, H L′, or M O′. The plane for the plan being parallel to the M P Q R face (here the top) of Z, the plan of that face is of the same shape as the face. The round-cornered rectangle A′ F′ G′ D′ B′ C′ of Fig. 29 is the plan to dimensions. For the same reason the plan of the O A T S face (here the base) is of the same shape as that face. The round-cornered rectangle A F G D B C of Fig. 29 is the plan to dimensions. How actually to draw these plans we shall deal with presently as a problem. The two circles constituting the plan of the frustum were concentric, that is, symmetrically disposed with respect to one another, because the frustum was an equal-tapering body; and the plans of top and base of the body we are now dealing with are symmetrical to each other for the same reason. The two plans (Fig. 29) together are the plan of the body Z.

Fig. 23*b*.



(26.) Looking at A B C D A′ B′ C′ D′ of the skeleton drawing (Fig. 23*b*) as at the plan of Z, we have, just as with the cone frustum, in the point A′ the plan of F, in the point B′ the plan of G, in the point C′ the plan of E, in the point L′ the plan of H, and in the point O′ the plan of M. Further as in the case of the frustum, if we join any point in the plan of the base, as A, to the plan of its corresponding point A′, then we have a right-angled triangle, F A A′, of which the hypotenuse F A represents the slant of the body, F A′, one of the sides containing the right angle, its height, and A′ A, the other side containing the right angle, the distance between the plans of the corresponding points F and A, which is also the distance between B and B′, C and C′, L and L′, O and O′, and between plans of corresponding points of the body anywhere, the body being of equal taper. As with Fig. 21*b* what points, in the plan, are the plans of corresponding points is clear from the fig. Where the plan of the body consists of straight lines, the plans of corresponding points are always the extremities of lines joining these straight lines perpendicularly; the extremities of A A′, B B′, C C′, and L L′, for instance. Where the plan of the body consists of arcs, the plans of corresponding points (compare with cone frustum) are the extremities of lines joining the arcs, and which, produced, will pass through the centre from which the arcs are described; the line O O′ for instance. To make all this quite plain, reference should again be made to Fig. 29; also to Fig. 28, which is the plan of an equal-tapering body with top and base parallel, and having flat sides, and semicircular ends. In Fig. 29, A A′, B B′, C C′, D D′, are lines joining the plan lines of the flat sides and ends perpendicularly, and the extremities of each of these lines are plans of corresponding points, that is to say, A and A′ are plans of corresponding points, as are also B and B′, C and C′, and D and D′. Also F and F′ are plans of corresponding points, being the extremities of the line F F′ which is a line joining the ends of the arcs which are the plans of one of the quadrant corners of the body. Similarly G and G′ are plans of corresponding points. In Fig. 28, F F′, G G′, D D′, E E′, are lines joining perpendicularly the plan lines of the flat sides of the body at their extremities where the semicircular ends begin; and F and F′, G and G′, D and D′, E and E′ are plans of corresponding points. Also A and A′ are plans of corresponding points, and B and B′, seeing that the lines joining these points, produced, pass respectively through O and O′, the centres from which the semicircular ends are described.

In the cone frustum, the actual distance between the plans of corresponding points was, we saw, equal to half the difference of the diameters of the two circles constituting its plan. Similarly with the body Z of Fig. 23*a*, and indeed with any equal-tapering body of which the top and base are parallel, if we have the lengths of the top and base given, or their widths, the distance between the plans of corresponding points (number of inches 'out of flue') is always equal to half the difference between the given lengths or widths. Thus, the distance between the plans of corresponding points of Z is equal to half the difference between A B and A′ B′ (Fig. 29) or between C D and C′ D′; and the distance between the plans of corresponding points of the body of which Fig. 28 is the plan, is equal to half the difference between A B and A′ B′ of that fig., or between F D and F′ D′.

Summarising we have

a. In the plans of equal-tapering bodies which have their tops and bases parallel, there is, all round, an equal distance between the plans of corresponding points of the tops and bases.

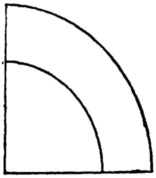
b. Conversely.—If, in the plan of a tapering body with top and base parallel, there is an equal distance all round between the plans of corresponding points of the top and base, then the tapering body is an equal-tapering body, that is, has an equal inclination of slant all round.

c. The plan of a round equal-tapering body having top and base parallel, consists of two concentric circles. The plan of a portion of a round equal-tapering body having top and base parallel, consists of two arcs having the same centre.

The corners of the body Z (Fig. 23*a*) are portions (quarters) of a round equal-tapering body; their plans are arcs (quadrants) of circles having the same centre

d. Conversely.—If the plan of a tapering body having top and base parallel, consists of two concentric circles, then the body is a frustum of a right cone. Also if the plan of a tapering body having top and base parallel, consists of two arcs having the same centre, then the body is a portion of a frustum of a right cone.

Fig. 24.



The plan of each end of the tapering body represented in plan in Fig. 28 consists of two arcs (semicircles) having the same centre; the ends are portions (halves) of a frustum of a right cone. The plan of each corner of the tapering body Z (Fig. 23*a*) consists of two arcs (quadrants, Fig. 29) having the same centre; the corners are portions (quarters) of a frustum of a right cone. The fig. annexed represents a quadrant corner in plan separately.

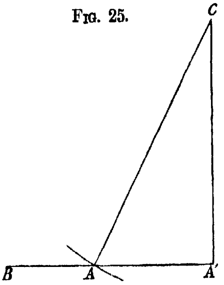
We conclude the chapter with some problems.

PROBLEM XII.

Given the height and slant of an equal-tapering body with top and base parallel; to find the distance between the plans of corresponding points of the top and base (number of inches 'out of flue').

Let C A′ (Fig. 25) be the given height. Draw A′B perpendicular to A′C; with C as centre and the given slant as

Fig. 25.



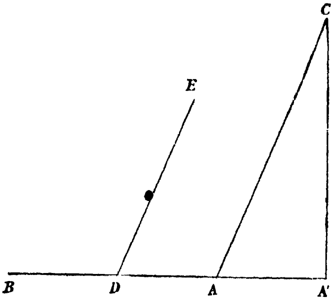
radius, describe an arc cutting B A′ in A. Then A A′ is the distance required.

PROBLEM XIII

Given the height of an equal-tapering body with top and base parallel, and the inclination of slant (number of inches 'out of flue'); to find the distance between the plans of corresponding points of the top and base.

Let C A′ (Fig. 26) be the given height. Through A′ draw a line A′ B perpendicular to C A′; from any point, D, in A′ B draw a line D E making with A′ B an angle equal to that of the given inclination. From C draw C A parallel to E D and cutting A′ B in A; then A A′ is the distance required.

FIG. 26.



PROBLEM XIV.

To draw the plan of a round equal-tapering body with top and base parallel (frustum of right cone), the diameter of either end being given and the height and slant.

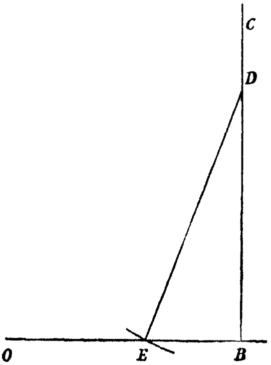
Case 1.—Given the height and slant and the diameter of the smaller end.

On any line O B (Fig. 17) set off O C equal to half the given diameter, and from C draw C D perpendicular to O B, Mark off C E equal to the given height, and with E as centre and radius equal to the given slant, describe an arc intersecting O B in B; then C B will be the distance in plan between corresponding points anywhere in the frustum; that is to say (by c, p. 55) O C will be the radius for the plan of the smaller end of the frustum, and O B the radius for the plan of the larger end.

CASE II.—Given the height and slant and the diameter of the larger end

On any line O B (Fig. 27), set off O B equal to half the given diameter, and now work from B towards O instead of from O towards B; thus. From B draw B C perpendicular to O B. Mark off B D equal to the given height, and with D as centre and radius equal to the given slant, describe an arc intersecting O B in E; then B E will be the distance in plan of corresponding points anywhere in the frustum; that is to say (by *c*, p. 55) O B will be the radius for the plan of the larger end of the frustum, and O E the radius for the plan of the smaller end.

FIG. 27.



PROBLEM XV.

To draw the plan of a round equal-tapering body with top and base parallel (frustum of right cone), the diameter of either end being given, and the number of inches 'out of flue' (distance between plans of corresponding points).

CASE I.—Given the number of inches 'out of flue,' and the diameter of the smaller end.

The radius for the smaller circle of the plan will be half the given diameter; the radius for the larger circle of the plan will be this half diameter with the addition of the number of inches 'out of flue.'

CASE II.—Given the number of inches 'out of flue,' and the diameter of the larger end.

The radius for the larger circle of the plan will be half the given diameter; the radius for the smaller circle of the plan will be the half diameter less the number of inches 'out of flue.'

(27.) It should be noted that with the dimensions given in this problem, we can draw plan only, we could not draw a pattern. To do that we must also have height given, for a plan of small height and considerable inclination of slant is also the plan of an infinite number of other frusta (*plural* of frustum) of all sorts of heights and inclinations of slant.

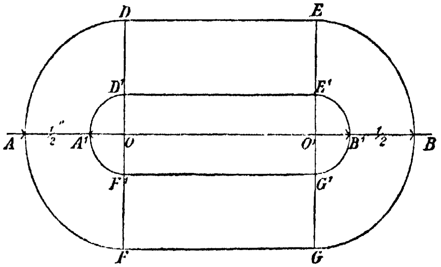
PROBLEM XVI.

To draw the plan of an oblong equal-tapering body with top and base parallel, and having flat (plane) sides and semicircular ends.

CASE I.—Where the length and width of the top are given, and the length of the bottom.

Commencing with the plan of the top, we know from § 25 that it will be of the same shape as the top; we have therefore to draw that shape. On any line A B (Fig. 28) mark off A B equal to the given length of the top. From A set off A O, and from B set off B O' each equal to half the given width of the top. Through O and O' draw lines perpendicular to A B; and with O and O' as centres and O A or O' B as radius describe arcs meeting the perpendiculars in D F and E G. As D F and E G pass through the centres O and O' respectively they are diameters, and the arcs are semicircles; these diameters, moreover, are each equal to the given width. Join D E, F G, and the plan of the top is complete.

Fig. 28.



The plan of the base will be of the same shape as the base, and we will suppose it smaller than the top. What we have then to do is to draw a figure of the same shape as the base, and to so place it in position with the plan of the top that we shall have a complete plan of the body we are dealing with. By *a*, p. 55, we know that the distances between the plans of corresponding points of the top and base all round the full plan will be equal. We have therefore first to ascertain the distance between the plans of any two corresponding points. This by § 26 will in the present instance be equal to half the difference between the given lengths of the top and base. Set off this half-difference, as the base is smaller than the top, from A to A′. Then with O and O′ as centres, and O A′ as radius, describe the semicircles D′ A′ F′, E′ B′ G′. Join D′ E′, F′ G′, and we have the required plan of the body.

Case II.—Where the length and width of the top are given, and the height and slant, or the height and the inclination of the slant (number of inches 'out of flue').

First draw the plan of the top as in Case I. Then if the height and slant are given, find by Problem XII. the distance between the plans of corresponding points. If the height and inclination of slant are given, find the distance by Problem XIII. If the inclination of the slant is given in the form of 'out of flue,' the number of inches 'out of flue' is the required distance. Set off this distance from A to A′ in the fig. of Case I., and complete the plan as in Case I.

Case III.—Where the length and width of the base (bottom) are given, and the height and slant, or the height and the inclination of the slant.

On any line A B (Fig. 28) mark off A′ B′ equal to the given length of the bottom. From A′ set off A′ O and from B′ set off B′ O′ each equal to half the given width of the bottom. Through O and O′ draw indefinite lines D F, E G perpendicular to A B; and with O and O′ as centres, and O A′ as radius describe the semicircles F′ A′ D′, G′ B′ E′, join D′ E′, F′ G′, and we have the plan of the bottom. Now by Problem XII. or Problem XIII., as may be required, find the distance between the plans of corresponding points, or take the number of inches 'out of flue,' if this is what is given. Set off this distance from A′ to A. With O and O′ as centres and O A as radius describe semicircles meeting the perpendiculars through O and O′ in D and F and in E and G. Join D E, F G, and the plan of the body is completed.

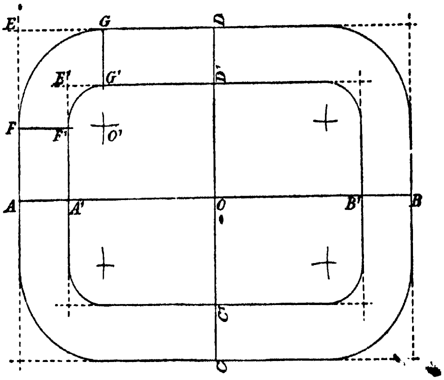
PROBLEM XVII.

To draw the plan of an oblong equal-tapering body with top and base parallel, and having flat sides, flat ends, and round (quadrant) corners.

CASE I.—Where the length and width of top and bottom (base) are given.

Draw any two lines A B, C D (Fig. 29) perpendicular to each other and intersecting in O. Make O A and O B each equal to half the length of the top, which we will suppose larger than the bottom, and O A′ and O B′ each equal to half the length of the bottom. Also make O C and O D each equal to half the width of the top, and O C′ and O D′ each equal to half the width of the bottom. Through C, D, C′, and D′ draw lines parallel to A B, and through A, B, A′, and B′ draw lines parallel to C D and intersecting the lines parallel to A B. We have now two rectangles or oblongs, and we require to draw the round corners, which are quarters of circles.

FIG. 29.



From the intersecting point E along the sides of the rectangle mark equal distances E F and E G, according to the size of quadrant corners required. With F and G as centres and E F, or E G as radius, describe arcs intersecting in Q′; and with O′ as centre and same radius, describe the arc F G, which will be a quadrant because if the points F and G be joined to O′ the angle F O′ G will be a right angle (p. 21). Draw F F′ parallel to A B and G G′ parallel to C D, and with O′ as centre and radius O′ F′ describe the arc F′ G′, which also will be a quadrant. We have now the plan of one of the quadrant corners; the other corners can be drawn in like manner.

(27*a*.) It is important to notice that the larger corner determines the smaller one. In practice it is therefore often best to draw the smaller corner first, otherwise it may sometimes be found, after having drawn the larger corner, that it is not possible to draw the smaller curve sufficiently large, if at all. To draw the smaller corner first, mark off from the intersecting point E′ equal lengths E′ F′, E′ G′, according to the size determined on for the corner. With F′ and G′ as centres and E′ F′ or E′ G′ as radius describe arcs intersecting in O′. Then O′ will be the centre for the smaller corner. It will also be the centre for the larger corner, which may be described in similar manner to the smaller corner in the preceding paragraph.

Case II.—Where the dimensions of the top are given and the height and slant, or the height and the inclination of the slant.

Draw the plan of the top, A D B C. Find the distance between the plans of corresponding points of the top and base by Problem XII., or Problem XIII., according to what is given; and set off this distance, as the base is smaller than the top, from A and B inwards towards O on the line A B, and from D and C inwards towards O on the line D C. Complete the plan by the aid of what has already been explained.

Case III.—Where the dimensions of the bottom are given, and the length and slant, or the height and the inclination of the slant.

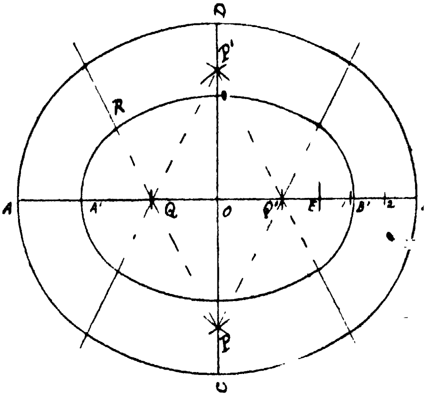
Draw the plan of the bottom, A′ D′ B′ C′, find the distance between the plans of corresponding points of top and bottom, set this off outwards from A′, D′, B′, and C′ and complete the plan by aid of what has already been stated.

PROBLEM XVIII.

To draw the plan of an oval equal-tapering body with top and base parallel, the length and width of the top and bottom being given.

Draw (Fig. 30) any two lines A B, C D intersecting each other at right angles in O. Make O A and O B each equal to half the given length of the larger oval (top or bottom, as may be), and O C and O D each equal to half its given width. A B and C D will be the *axes* of the oval. From A, on A B,

Fig. 30.



mark off A E equal to C D the width of the oval, and divide E B into three equal parts. With O as centre and radius equal to two of these parts, as from E to 2, describe arcs cutting A B in Q and Q′. With Q and Q′ as centres and Q Q′ as radius describe arcs intersecting in P and P′; and from P and P′ draw lines of indefinite length through Q and Q′. With P and P′ as centres and radius P D describe arcs (the side arcs), their extremities terminating in the lines drawn through Q and Q′; and with Q and Q′ as centres, and radius Q A, describe arcs (the end arcs) to meet the extremities of the side arcs. This completes the plan of the larger oval.

To draw the plan of the smaller oval. Make O A′ and O B′ each equal to half the length of the smaller oval, and with Q and Q′ as centres and Q A′ as radius describe the end curves, their extremities terminating, as do the outer end-curves, in the lines drawn through Q and Q′; the point R is an extremity of one of the smaller curves. With P and P′ as centres and radius P R, describe the side curves. The plan of the oval equal-tapering body is then complete; of which either the larger or smaller ovals are plan of top and bottom according to the purpose the article may be required for.

(28.) The plans of corresponding points in the plan of an oval equal-tapering body will be the extremities of any line joining the inner and outer curves anywhere, and that, produced, will pass through the centre from which the curves where joined by the line are described.

To draw the plan of an oval equal-tapering body with top and base parallel, other dimensions than the above may be given. For instance the top or bottom may be given, and either the height and slant, or the height and the inclination of the slant (number of inches out of flue). It will be a useful practice for the student to work out these cases for himself by the aid of the instruction that has been given.

CHAPTER VI.

PATTERNS FOR ARTICLES OF EQUAL TAPER OR INCLINATION OF SLANT, AND HAVING FLAT (PLANE) SURFACES.

(CLASS I. *Subdivision b.*)

DEFINITION.

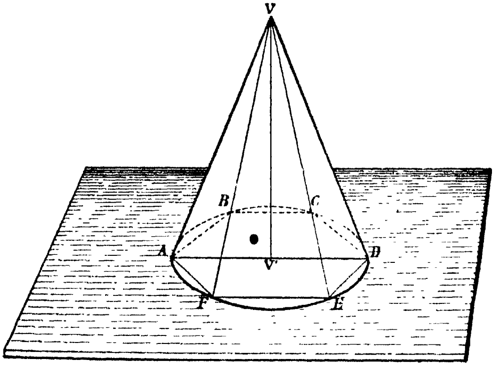
(29.) Pyramid.—A pyramid is a solid having a base of three or more sides and triangular faces meeting in a point above that base, each side of the figure forming the base being the base of one of the triangular faces, and the point in which they all meet being the *apex.* The shape of the base of a pyramid determines its name; thus a pyramid with a triangular base is called a *triangular* pyramid; with a square base, a *square* pyramid; with a hexagon base, an *hexagonal* pyramid (Fig. 31); and so on. The centre of the base of a pyramid is the point in which perpendicular lines bisecting all its sides will intersect. If the apex of a pyramid is perpendicularly above the centre of its base, the pyramid is a *right* pyramid (Fig. 31 represents a right pyramid), in which case the base is a regular polygon and the triangular faces are all equal and all equally inclined. In a pyramid, the line joining the apex to the centre of the base is called the *axis* (the line V V′, Fig. 31) of the pyramid.

(30.) An important property that a right pyramid possesses is that it can be *inscribed in a right cone.*

(31.) A pyramid is said to be inscribed in a cone when both the pyramid and the cone have a common apex, and the base of the pyramid is inscribed in the base of a cone; in other words, when the angular points of the base of the pyramid are on the circumference of the base of the cone and the apex of cone and pyramid coincide.

(32.) Fig. 31 shows a right pyramid inscribed in a right cone. The apex V is common to both pyramid and cone, and the A, B, C, &c., of the base of the pyramid are on the circumference of the base of the cone. Also the axis V V′ is common to both cone and pyramid. Further, the edges V A, V B, V C, &c., of the pyramid are lines on the surface of the cone, such lines or edges being each a slant of the cone, or in other words a radius of the pattern of the cone in which the pyramid is inscribed. It hence follows, that if the pattern of the cone in which a right pyramid is inscribed be set out with the lines of contact of cone and pyramid, as V A, V B, &c., on it, and the extremities of these lines be joined, we shall have the pattern for the pyramid. Thus, the drawing a pattern for a right pyramid resolves itself into first determining the cone which circumscribes the pyramid, and next drawing the pattern of that cone with the lines of contact of pyramid and cone upon it.

FIG. 31.



PROBLEM XIX.

To draw the pattern for an hexagonal right pyramid, its height and base being given.

Draw (Fig. 32*a*) the plan A B C D E F of the base of the pyramid, which will be of the same shape as the base (*see* Chap. V.); the base in fact will be its own plan. Next draw any two lines O A, B A (Fig. 32*b*), perpendicular to each other and meeting in A; make A B equal to the radius of the circumscribed circle (Fig. 32*a*), and A O equal to the given height of the pyramid. Join B O; then B O is a slant of the cone in which the pyramid can be inscribed, that is to say, is a radius of the pattern of that cone. The line B O is also a line of contact of the cone, in which the pyramid can be inscribed, that is, is one of the edges of the pyramid.

FIG. 32*a*.

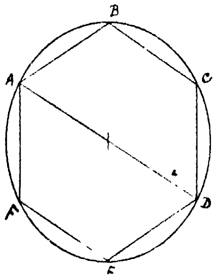
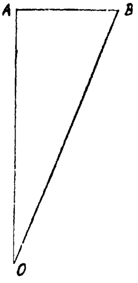


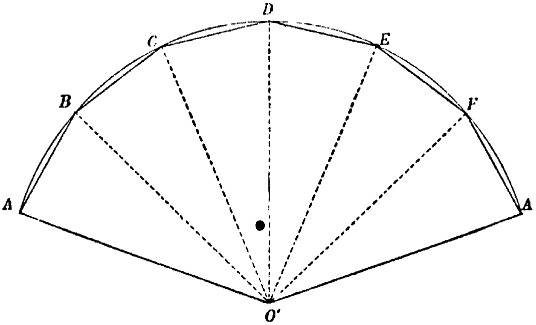
FIG. 32*b*.



To draw the pattern (Fig. 32*c*). With any point O′ as centre and B O (Fig. 32*b*) as radius, describe an arc A D A, and in it take any point A. Join A O′, and from A mark off A B, B C, C D, D E, E F, and F A, corresponding to A B, B C, C D, D E, E F, and F A of the hexagon of Fig. 32*a*, and join the points B, C, D, E, F and A to O′. Join A B, B C, C D, D E, E F, and F A, by straight lines; and the figure bounded by O′ A, the straight lines from A to A, and A O′, will be the pattern required. The lines B O′, C O′, &c., correspond to the edges of the pyramid, and show the lines on which to 'bend up' to get the faces of the pyramid, the lines O′ A and O′ A then butting together to form one edge.

Similarly the pattern for a right pyramid of any number of faces can be drawn, the first step always being to draw the plan of the base of the pyramid; the circle passing through the angular points of which will be the plan of the base of the cone in which the pyramid can be inscribed.

FIG. 32c.



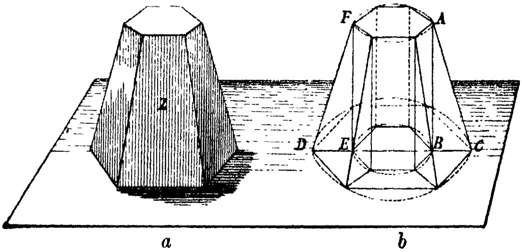
Suppose, instead of the dimensions from which to draw the plan of the base of the pyramid, the actual plan be given. The centre from which to strike the circumscribing circle can then be found by the Definition § 29.

DEFINITION.

(33.) Truncated pyramid. Frustum of pyramid.—If a pyramid be cut by a plane parallel to its base, the part containing the apex will be a complete pyramid, and the other part will be a tapering body, the top and base of which are of the same shape but unequal. This tapering body is called a *truncated pyramid*, or a *frustum of a pyramid*. The faces of a truncated pyramid which is a frustum of a right pyramid are all equally inclined. In Fig. 33 is shown such a frustum standing on a horizontal plane.

(34.) Comparison should here be made between this definition and that of a frustum of a cone (*see* § 12), which it closely follows; also between Fig. 21 and Fig. 33.

FIG. 33.



(35.) Articles of equal taper or inclination of slant and having flat (plane) surfaces and top and base parallel (hexagonal coffee-pots; hoods; &c.), are portions of right pyramids (truncated pyramids), or portions of truncated pyramids.

(36.) Exactly as a pyramid can be inscribed in a cone, so a truncated pyramid can be inscribed in a frustum of a cone, and the edges of the truncated pyramid are lines on the surface of that frustum. The skeleton drawing, Fig. 33*b*, shows a right truncated pyramid inscribed in a cone frustum. It also represents the plan of the cone frustum, and that of the pyramid frustum, with the lines of projection (*see* Chapter V.), of the smaller end of the latter on to the horizontal plane. This inscribing in a cone gives an easy construction for setting out the pattern of a truncated pyramid; which construction is, to first draw the pattern for the pyramid of which the truncated pyramid is a portion; and then mark off on this pattern the pattern for the pyramid that is cut off. Here again comparison should be made with what has been stated about the pattern of a frustum of a cone (*see* § 16), and the resemblance noted.

PROBLEM XX.

To draw the pattern for an equal-tapering body made up of flat surfaces (truncated right pyramid), the height, and top and bottom being given.

Suppose the equal-tapering body to be hexagonal.

To draw the required plan of the frustum. The plans of the top and bottom are respectively of the same shape as the top and bottom (§ 25). Draw (Fig. 34*a*) A B C D E F the larger hexagon (Problem X., Chap. II.) and its diagonals

FIG. 34*a*.

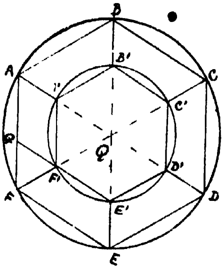
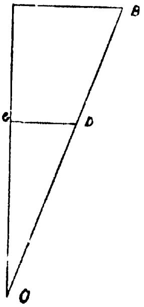


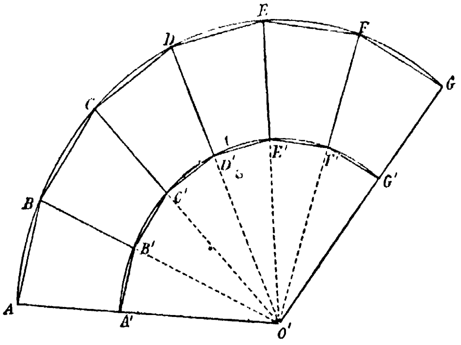
FIG. 34*b*.



A D, B E, C F. intersecting in Q. On any one of the sides of this hexagon mark off the length of a side of the smaller hexagon, as A G on A F, and through G draw G F' parallel to the diagonal A D, and cutting the diagonal F C in F'. With Q as centre and Q F' as radius describe a circle. The points in which this cuts the diagonals of the larger hexagon will be the angular points of the smaller hexagon. Join each of these angular points, beginning at F', to the one next following, as F'E', E'D', &c. Then F'E'D'C'B'A' is the plan of the smaller hexagon, and, so far as needed for our pattern, the plan of the 'equal-tapering body made up of flat surfaces' is complete. The lines A A', B B', C C', will be the plans (see and compare lines D E and B C of Fig. 33*b*) of the slanting edges of the frustum.

Next draw (Fig. 34*b*) two lines O A, B A, perpendicular to each other, and meeting in A; make A B equal to the radius of the circle circumscribing the larger hexagon of plan, and

Fig. 34*c*.



A C equal to the given height of the body. Through C draw C D perpendicular to A O, and make C D equal to the radius of the circle which passes through the angular points of the smaller hexagon (Fig. 34*a*). Join B D, and produce it to meet A O in O.

To draw the required pattern (Fig. 34*c*). Draw any line O'A, and with O' as centre and O B, O D (Fig. 34*b*) as radii, describe arcs A D G, A'D'G' (Fig. 34*c*). Then take the straight line length A B (Fig. 34*a*), and set it off as a chord from A (Fig. 34*c*) on the arc A D G. Do the same successively, from point B, with the straight line lengths (Fig. 34*a*) B C, C D, D E, E F, F G, the terminating point of each chord as set off, being the starting point for the next, the chord F G (Fig. 34*c*) corresponding to the straight line length F A (Fig. 34*a*). Join (Fig. 34*c*) the points B, C, D, E, F and G to O′ by lines cutting the arc A′ D′ G′ in B′, C′, D′, E′, F′, and G′. Join A B, B C, C D, &c., and A′ B′, B′ C′, C′ D′, &c., by straight lines; then A D G G′ D′ A′ is the pattern required.

The frustum of pyramid is here hexagonal, but by this method the pattern for any regular pyramid cut parallel to its base can be drawn. The next problem will show methods for larger work.

(37.) If O′ A D G (Fig. 34*c*), the pattern for the cone in which the frustum of pyramid is inscribed, be cut out of zinc or other metal, and small holes be punched at the points A B, C, &c., and A′, B′, C′, &c ; and if the cone be then made up with the lines O′ A, O′ B′, &c., marked on it inside, and wires be soldered from hole to hole successively to form the top and bottom of the truncated pyramid, then (1) the whole pyramid of which the truncated pyramid is a portion, (2) the pyramid that is cut off, as well as (3) the truncated pyramid, will be clearly seen inscribed in the cone. The making such a model will amply repay any one who desires to be thoroughly conversant with the construction of articles of the kind now under consideration.

PROBLEM XXI.

To draw, without using long radii, *the pattern for an equal-tapering body made up of flat surfaces, the height and top and bottom being given.*

Again suppose the body to be hexagonal.

Case I. For ordinarily large work.

Draw (Fig 35*a*) the plan as by last problem. Next join A B′, and draw B′ G perpendicular to A B′ and equal to the given height of the body. Also draw B′ H perpendicular to, B B′; and equal to the given height. Join A G and B H; then A G and B H are the true lengths of A B′ and B B′ respectively.

FIG. 35a.

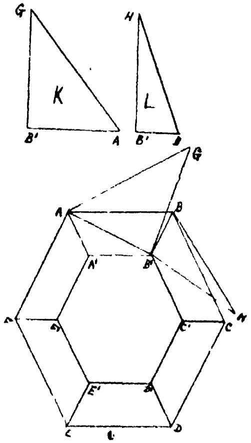
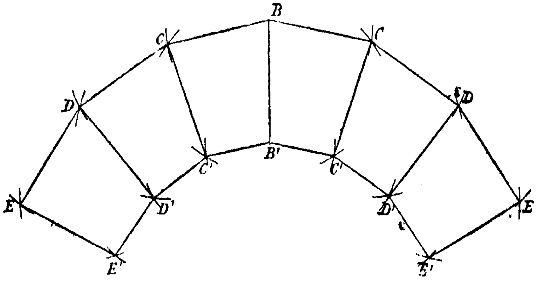


FIG. 35b.



To draw the pattern. Draw (Fig. 35b) B B′ equal to B H (Fig. 35a). With B as centre and radius equal to B A

(Fig. 35*a*), and with B′ as centre and radius equal to G A (Fig. 35*a*) describe arcs intersecting in C, right and left of B B′. With B as centre and the same G A as radius, and with B′ as centre and radius B′ A′, describe arcs intersecting in C′, right and left of B′ B. Join B C, C C′, C′ B′ (Fig. 35*b*); then B B′ C′ C is the pattern of the face B B′ C′ C in plan (Fig. 35*a*). The other faces C D C′ D′, B C B′ C′, &c. (Fig. 35*b*) of the frustum are described in exactly similar manner, G A being the distance between diagonally opposite points of any face as well as of the face B B′ C′ C. The triangles A B′ G, B B′ H, can be drawn apart from the plan, as shown at K and L.

(38.) It should be observed that if the pattern is truly drawn, the top and bottom lines of each face will be parallel, as B C, B′ C′, of face B C B′ C′ (Fig. 35 *b*); and that this gives an easy method of testing whether the pattern has been accurately drawn.

Case II.—For very large work; where it is inconvenient to draw the whole of the plan.

Draw A B (Fig. 36*a*) equal in length to the end-line of one of the faces of the frustum at its larger end, and produce it both ways. With B as centre and radius B A describe a semicircle, which divide into as many equal parts as the frustum has faces (Problem IX.). Here it is hexagonal, and the points of division working from point 1, are 2, C, 4, &c. Through the second division point, here C, draw a line to B, then A B C is the angle made in plan by two faces of the frustum one with another, and A B, B C are two adjacent end-lines of the plan of its larger ends. Bisect the angle A B C (Problem VIII.) by B E; and draw a line C C′ from C making the angle C′ C B equal to the angle C B E (Problem I.). On B C set off B D equal to the end-line of one of the faces of the frustum at its smaller end; and draw D C′ parallel to B E, cutting C C′ in C′. Through C′ draw C′ B′ parallel to C B and meeting B E in B′; and draw B′ A′ parallel to B A and equal to B D or C′ B′. Join A′ A, and we have in A′ A B C C′ the plan of two adjacent faces of the tapering body or truncated pyramid. Next from B′ let fall B′ G perpendicular to A B, and make G F equal to the given height of the frustum. Join F B′, then F B′ is the true length of B′ G. Through B′ draw B′ H perpendicular to B′ B and equal to the given height. Join H B, then H B is the true length of B B′ one of the edges of the frustum.

To draw the pattern. Draw (Fig. 36*b*) B B′ equal to H B (Fig. 36*a*). With B and B′ as centres and radii respectively B G and F B′ (Fig. 36*a*) describe arcs intersecting in G (Fig. 36*b*). Join B G and produce it making B A equal to B A (Fig. 36*a*). Through B′ draw B′ A′ parallel to B A and equal to B′ A′ (Fig. 36*a*), and join A A′; then B A A′ B′ will be the pattern of one face of the frustum. The adjacent face B C C′ B′ is drawn in similar manner. The fig. shows the pattern for two faces only of the equal-tapering body, because in cases where this method would have to be employed, two faces are probably the utmost that could be cut out in one piece. Sometimes each face would have to be cut out separately, or perhaps even one face would have to be in portions. *Any point* in B′ A′ (Fig. 36*a*) instead of B′ can be chosen from which to let fall a perpendicular to B A, and the true length of B′ G found as explained. The choice of position depends upon the means at hand for drawing large

FIG. 36*a*.

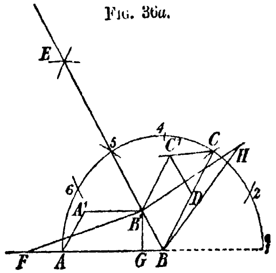
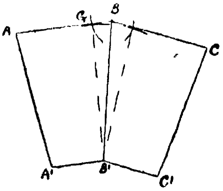


FIG. 36*b*.



arcs, the radius of the arc B′ G (Fig. 36*b*) increasing in length as the point G approaches nearer to A. If the pattern be truly drawn, B A will be perpendicular to B′ G.

It must not be forgotten that these methods are, both of them, quite independent of the number of the sides of the pyramid. Also it should be noted that B E does not of necessity pass through a division point, nor of necessity is C C′ parallel to A B. These are coincidences arising from the frustum being here hexagonal.

PROBLEM XXII.

To draw the pattern for an oblong or square equal-tapering body with top and base parallel, and having flat sides and ends. (The bottom is here taken as part of the body, and the whole pattern is in one piece.)

Note.—This problem will be solved in the problem next following. We adopt this course because the article there treated of is so important an example of the oblong equal-tapering bodies in question, that it is desirable to make that, the special problem, the primary one, and this, the general problem, secondary to it. Its solution will be found at the end of Case I.

PROBLEM XXIII.

To draw the pattern for a baking-pan.

A baking-pan has not only to be water-tight, but also to stand heat; hence when made in one piece the corners are seamless.

Case I.—Where the length and width of the bottom, the width of the top, and the slant are given.

Draw two lines X X, Y Y, intersecting at right angles in O (Fig. 37); make O A′ and O B′ each equal to half the length of the bottom, and O C′ and O D′ each equal to half its width. Through C′ and D′ draw lines parallel to Y Y; also through A′ and B′ draw lines parallel to X X and inter-

Fig. 37.

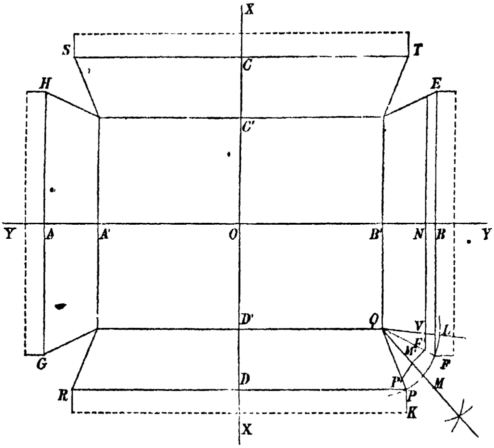
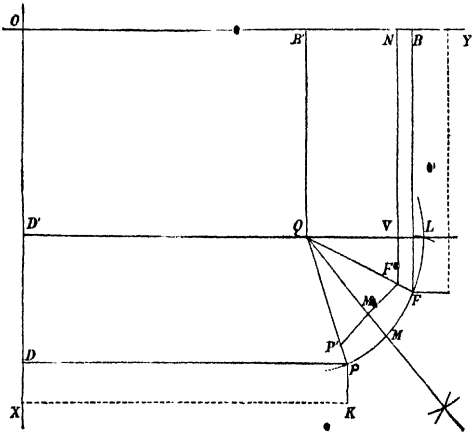


Fig. 38.



(Quarter of Fig. 37, enlarged.)

secting the lines drawn through C′ and D′; we get by this a rectangular figure, which is the shape of the bottom. Make A′A, B′B, C′C, and D′D each equal to the given slant; through B and A draw E F and G H parallel to X X; through C and D draw S T and R P parallel to Y Y; and make B E, B F, A H, and A G each equal to half the given width of the top. Join Q F and with Q, which is one of the angular points of the bottom, as centre and radius Q F describe an arc cutting P R in P. Join Q P (the working can here be best followed in Fig. 38), bisect the angle F Q P by Q M (Problem VIII., Chap. II.); and draw a line Q L making with F Q an angle equal to the angle F Q M. The readiest way of doing this is by continuing the arc P F to L, then setting off F L equal to F M, and joining L Q. Now on B B′ set off B N equal to the thickness of the wire to be used for wiring, and through N draw N F″ parallel to E F (Fig. 37) and cutting Q L and Q F in V and F″; make Q P′ equal to Q F″, and Q M′ equal to Q V; and join P′ M′ and M′ F″. Repeat this construction for the other three corners and the pattern will be completed. This is not done in Fig. 37, for a reason that will appear presently. We shall quite understand the corners if we follow the letters of the Q corner in Fig. 38. These are B F F″ M′ P′ P. The dotted lines drawn outside the pattern (Fig. 37) parallel to E F, P R, G H, and S T show the allowance for fold for wiring. It is important that the ends of this allowance for fold shall be drawn, as, for instance, P K (both figs.), perpendicular to their respective edges.

Now as to bending up to form the pan. When the end adjacent and the side adjacent to the corner Q come to be bent up on B′ Q and D′ Q so as to bring F Q and P Q into junction, it is evident that as F Q and P Q are equal we shall obtain a true corner. To bring F Q and P Q into junction it is likewise manifest that the pattern will have also to be bent on the lines F″ Q, P′ Q, and M′ Q. This fold on each other of P′ Q M′ and F″ Q M′ is generally still further bent round against Q V.

(39.) The truth of the pan when completed, and the ease with which its wiring can be carried out, depend entirely on the accuracy of the pattern at the corners. This must never be forgotten. In marking out a pattern, only one corner need be drawn, as the like to it can be cut out separately and used to mark the remaining corners by. The points R, S, and T, the fixing of which will aid in this marking out, can be readily found, thus:—For the R corner to come up true it is clear that D R will have to be equal to D P, from which we learn that D P is half the length of the top. Having then determined the point P we have simply to set off D R, C S, and C T each equal to D P.

If the pattern Fig. 37 were completed with the corners as at R, S, and T, that would be the solution of Problem XXII., and the ends and sides being bent up, we should get an oblong equal-tapering body with top and bottom parallel and having flat sides and ends.

Case II.—Where the length and width of the bottom, the length of the top, and the slant are given.

The only difference between this case and the preceding is that D P (Fig. 37), half the length of the top, is known instead of B F the half-width of top. To find the half-width of top; with Q as centre and Q P as radius describe an arc cutting B F in F. Then B F will be half the width of the top, just as (§ 39, previous case) we saw that D P was half the length of the top. The remainder of the construction is now as in Case I.

(40.) It will be evident that, in this problem, choice of dimensions is not altogether arbitrary. The lines Q F and Q P the meeting of which forms the corner, must always be of the same length. This limits the choice; for with the dimensions of bottom given, and the slant, and the width of the top, the length of the top cannot be fixed at pleasure, but must be such as will bring Q F and Q P into junction; and *vice versâ* if the length of the top is given. It is unnecessary to follow the limit with other data.

CASE III.—Where the length and width of the top, and the slant and the height are given.

The data in this case and the next are the usual data when a pan has to be made to order. The difference between this case and the preceding is that the size of the bottom is not given, but has to be determined from the data. Excepting as to finding the dimensions of this, the case is the same as Cases I. and II. All that we have now to do is therefore to find the dimensions of the bottom, thus:—

FIG. 39.

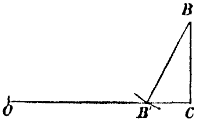
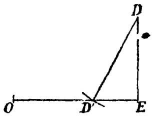


FIG. 40.



Draw O C (Fig. 39) equal to half the given length of the top, and through C draw C B perpendicular to O C and equal to the given height. With B as centre and radius equal to the given slant describe an arc cutting O C in B'. Then O B' is the required half-length of the bottom.

Next draw O E (Fig. 40) equal to half the given width of the top, and through E draw E D perpendicular to O E and equal to the given height. With D as centre and radius equal to the given slant describe an arc cutting O E in D'. Then O D' is the required half-width of the bottom.

CASE IV.—Where the length and width of the top and the length and inclination of the slant are given.

This is a modification of Case III.

Let B B' (Fig. 39) be the length of the given slant, and the angle that B B' make with B' C be the inclination of the slant. Through B draw B C perpendicular to B' C. Then half the given length of the top less B' C will be the required half-length of the bottom; and similarly half the given width of the top less B' C will be the required half-width of the bottom.

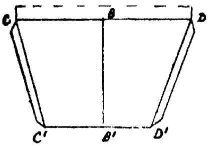
PROBLEM XXIV.

To draw the pattern for an equal-tapering body with top and base parallel, and having flat sides and ends (same as Problem XXII.), but with bottom, sides, and ends in separate pieces; the length and width of the bottom, the width of the top, and the slant being given.

To draw the pattern for the end.

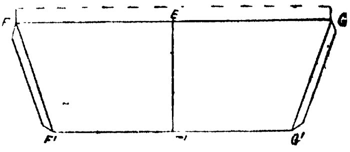
Draw B B′ (Fig. 41) equal to the given slant, and through B and B′ draw lines C D and C′ D′ perpendicular to B B′. Make B C and B D each equal to half the given width of the top; also make B′ C′ and B′ D′ each equal to half the given width of the bottom. Join C C′ and D D′; then C′ C D D′ will be the end pattern required.

FIG. 41.



END PATTERN.

FIG. 42.



SIDE PATTERN.

To draw the pattern for the side.

Draw E E′ (Fig. 42) equal to the given slant, and through E and E′ draw F G and F′ G′ perpendicular to E E′. Make E′ F′ and E′ G′ each equal to half the given length of the bottom, and with F′ and G′ as centres and radius D′ D (Fig. 41) describe arcs cutting F G in points F and G. Join F′ F and G′ G; then F′ F G G′ will be the side pattern required.

It should be noted that, as in the preceding problem, the width of the top determines the length of the top and *vice versâ*, also that lines such as D D′, G G′ must be equal.

(41.) If the ends of the body are seamed ('knocked up') on to the sides, as is usual, twice the allowance for lap shown in Fig. 41, must be added to the figure C C′ D′ D. Similarly if the sides are seamed on to the ends, a like double allowance for lap must be added to F F′ G′ G, instead of to the end pattern.

CHAPTER VII.

PATTERNS FOR EQUAL-TAPERING ARTICLES OF FLAT AND CURVED SURFACES COMBINED.

(CLASS I. *Subdivision c.*)

From what has been stated about the plans of equal-tapering bodies, and from *d*, p. 55, it will be evident that the curved surfaces of the articles now to be dealt with, are portions of frusta of cones.

PROBLEM XXV.

To draw the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and equal semicircular ends (an 'equal-end' pan, for instance), the dimensions of the top and bottom of the body and its height being given.

Four cases will be treated of; three in this problem and one in the problem following.

CASE I.—Patterns when the body is to be made up of four pieces.

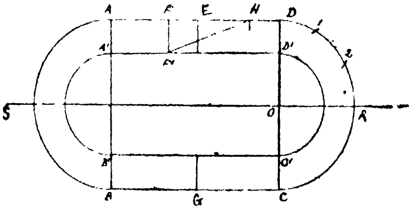
We may suppose the article to be a pan. Having drawn (Fig. 43) the plan A B C S A' D' C' B' by the method of Problem XVI. (as well as the plan, the lines of its construction are shown in the fig.), let us suppose the seams are to be at A, B, C, and D, where indeed they are usually placed. Then we shall require one pattern for the flat sides and another for the curved ends.

To draw the pattern for the sides.

Anywhere in A D (Fig. 43) and perpendicular to it draw

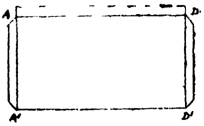
F F′, and make F H equal to the given height; join F′H; then F′H is the length of the slant of the article, and therefore the width of the pattern for the sides.

FIG. 43.



Draw A A′ (Fig. 44) equal to F′H (Fig. 43). Through A and A′ draw lines perpendicular to A A′. Make A D (Fig. 44) equal to A D (Fig. 43), and draw D D′ parallel to A A′. The rectangle A D D′ A′ will be the side pattern required. The fig. also shows extras for lap.

FIG. 44.



SIDE PATTERN.

To draw the pattern for the ends.

Draw (Fig. 45a) two lines D A, O A, perpendicular to one another and meeting in A; make D A equal to O D (Fig. 43), the larger of the radii of the semicircular ends; and on A O set off A G equal to the given height. Draw G D′ perpendicular to A G and equal to O D′ (Fig. 43), the smaller of the radii of the semicircular ends; join D D′ and produce it, meeting A O in O. Then with any point O′ (Fig. 45b), as centre, and O D (Fig. 45a) as radius, describe an arc D C, and with the same centre and radius O D′ (Fig. 45a) describe an arc D′ C′. Draw any line D O′, cutting the arcs in points D and D′. Divide D R (Fig. 43) into any number of equal parts, say three. From D (Fig. 45b) mark off these three

FIG. 45a.

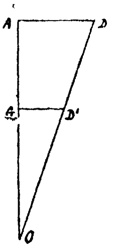
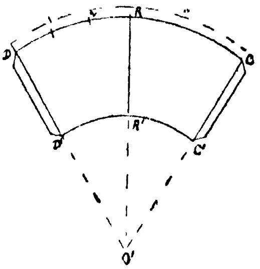


FIG. 45b.



dimensions to R, make R C equal to D R, and join C O′; then D C C′ D′ will be the end pattern required.

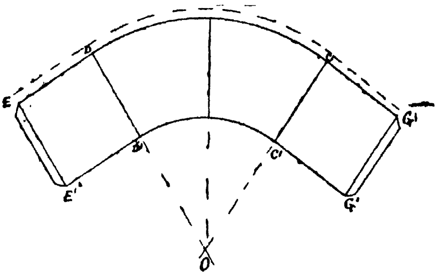
If R be joined to O′, then D D′ R′ R will be half the pattern. The centre line R R′ is very useful, because, in making up the article, the point R′ must meet the line S R (axis) of Fig. 43, otherwise the body will be twisted in consequence of the bottom not being true with the ends.

CASE II.—Pattern when the body is to be made up of two pieces.

Secondly, suppose the article can be made in two pieces (halves), with the seams at E and G (Fig. 43); the line E G (part only of it shown), being the bisecting line of the plan. It will be seen by inspection of the fig. that we require one pattern only, namely, a pattern that takes in one entire end of the article with two half-sides attached.

Draw the end pattern D D'C'C (Fig. 46) in precisely the same manner that it is drawn in Fig. 45*b*. Through C and C' draw indefinite lines C G C'G', perpendicular to C C', and through D and D' draw indefinite lines D E, D'E' perpendicular to D D'. Make D E, D'E' each equal to D E (Fig. 43) and join E F', also make C G, C'G', each equal to C G (Fig. 43), and join G G'. Then E E' G' G will be the pattern required.

Fig. 46.



Case III.—Pattern when the body can be made of one piece.

Thirdly, suppose the article can be made in one piece, with the seam at S (Fig. 43). Then evidently the pattern will be made up of the pattern of one entire end, side patterns attached to this, and half an end pattern attached to each side pattern.

Draw D D'C'C (Fig. 47) the end pattern. Through C and C' draw lines perpendicular to C C', and each equal to C B (Fig. 43); and join B B'. Produce B B', and with B as centre and O D (Fig. 45*a*), the larger of the radii for the end pattern, as radius, describe an arc cutting the produced line in O. With O as centre and O B and O B′ as radii respectively, draw arcs B S, B′S′. Make B S equal to D R (Fig. 47)

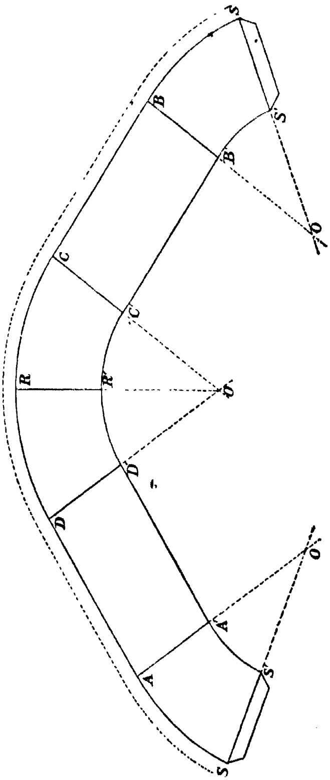


Fig. 47.

and join S O. Repeating this construction for D D′ A′ A the other side pattern, and A A′ S′ S the remaining half-end pattern, will complete the pattern required.

PROBLEM XXVI.

To draw, **without long radii,** *the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and equal semicircular ends; the dimensions of the top and bottom of the body and its height being given.*

This problem is a fourth case of the preceding, and exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

Draw half the plan (Fig. 48*a*). Divide D C into six or more equal parts, and join 1, 2, &c., to O, by lines cutting D′ C′ in 1′, 2′, &c., and join D 1′. Draw D E perpendicular to D 1′ and equal to the given height, and join E 1′. (The line 1 to 1′ appears to, but does not, coincide with E 1′.) Then E 1′ may be taken as the true length of D 1′. Next, producing as necessary, make D A equal to the given height. Joining D′ to A gives the true length of D D′.

FIG. 48*a*.

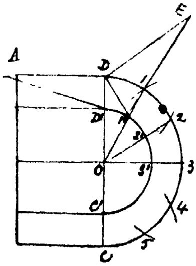
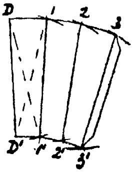


FIG. 48*b*.



To draw the end pattern. Draw Fig. (48*b*) D D′ equal to D′A (Fig. 48*a*). With D and D′ as centres, and radii respectively D 1 and E 1′ (Fig. 48*a*) describe arcs intersecting in point 1 (Fig. 48*b*). With D′ and D as centres and radii respectively D′ 1′ and E 1′ (Fig. 48*a*) describe arcs intersecting in point 1′ (Fig. 48*b*). By using points 1 and 1′ (Fig. 48*b*) as centres, instead of D and D′, and repeating the construction, the points 2 and 2′ can be found. Next, using points 2 and 2′ as centres and repeating the construction, find points 3 and 3′, and so on for the remaining points necessary to complete the end pattern, which is completed by joining the various points, as 3 to 3′ by a straight line; D, 1, 2, and 3, by a line of regular curve; and D′, 1′, 2′ and 3′, also by a line of regular curve. Only half the end pattern is shown in Fig. 48*b*. The side pattern can be drawn as shown in Fig. 44.

PROBLEM XXVII.

To draw the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and ends, and round corners (an oblong pan with round corners, for instance); the height and the dimensions of the top and bottom being given.

Again four cases will be treated of; three in this problem, and one in the problem following.

Case I.—Pattern when the body is to be made up of four pieces.

The plan of the article, with lines of construction, drawn by the method of Problem XVII., is shown in Fig. 49. We will suppose the seams are to be at P, S, Q, and R, that is, at the middle of the sides and ends. The pattern required is therefore one containing a round corner, with a half-end and a half-side pattern attached to it. The best course to take is to draw the pattern for the round corner first, which, as will be seen from the plan, is a quarter of a frustum of a cone.

Draw two lines (Fig. 50*a*) O A, C A, meeting perpendicularly to one another in A; make A C equal to O A (Fig. 49), the radius of the larger arc of the plan of one of the corners, and make A D equal to the given height of the body. Draw D E perpendicular to A D and equal to the radius of the smaller arc of the plan of a corner, and join C E and produce it to meet A O in O. With any point O′ (Fig. 50*b*) as centre and radius equal to O C (Fig. 50*a*), describe an arc A B, and with the same centre and radius equal to O E (Fig. 50*a*) describe an arc A′ B′. Draw any line A O′ cutting the arcs

FIG. 49.

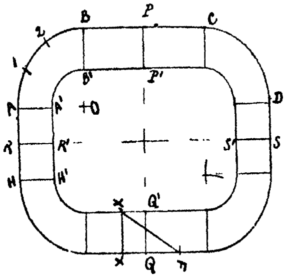


FIG. 50*a*.

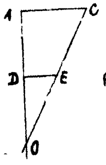
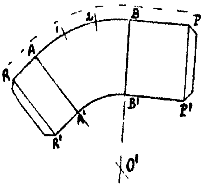


FIG. 50*b*.



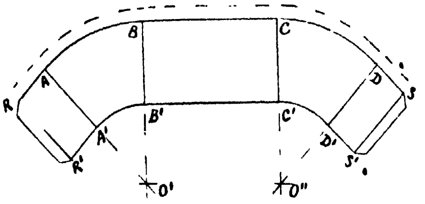
in the points A and A′, and make A B equal to A B (Fig. 49) by marking off the same number of equal parts along A B (Fig. 50*b*) that we divide (arbitrarily) A B (Fig. 49) into. Join B O′, cutting in B′ the arc A′ B′. Through B and B′ draw B P, B′ P′ perpendicular to B B′, make B P equal to B P (Fig. 49) and draw P P′ perpendicular to B P. Through A and A′ draw A R and A′ R′ perpendicular to A A′, make A R equal to A R (Fig. 49) and draw R R′ perpendicular to A R. Then R R′ P′ P will be the pattern of one round corner, with a half-end and a half-side pattern attached left and right.

CASE II.—Pattern when the body is to be made up of two pieces.

Now suppose the seams are to be at the middle of each end, at R and S (Fig. 49). The pattern required will then be of twice the amount shown in Fig. 50*b*. It will be found best to commence with the side pattern.

Draw in the plan (Fig. 49), any line X X perpendicular to the Q side-line; then X and X′ will be plans of corresponding points (§ 26). Make X F equal to the given height of the body and join X′ F. Then X′ F is the length of a slant of the body. Draw B B′ (Fig. 51) equal to X′ F (Fig. 49), and through B and B′ draw B C and B′ C′ perpendicular to B B′. Make B C equal to B C (Fig. 49) and draw

FIG. 51.



C C′ perpendicular to B C; then B C C′ B′ will be the pattern for the side.

We have now to join on to this, at B B′ and C C′, the patterns for the round corners, which can be done thus. Produce B B′ and C C′, and make B O′ and C O″ each equal to O C (Fig. 50*a*) the radius of the larger arc of the corner pattern. With O′ and O″ as centres and O′ B as radius, describe arcs B A and C D, and with the same centres and O′ B′ as radius, describe arcs B′ A′ and C′ D′. Make B A and C D equal each to B A in the plan (Fig 49), and join A C, cutting the arc B′ A′ in A′, and D O″ cutting the arc C′ D′ in D′. Through A and A′ draw A R and A′ R′ perpendicular to A A′; make A R equal to A R (Fig. 49), and draw R R′ perpendicular to A R. Next through D and D′ draw D S and D′ S′ perpendicular to D D′; make D S equal to D S (Fig. 49), and draw S S′ perpendicular to D S. Then R R′ S′ S will be the pattern required.

It should be noted that O′ B′ and O″ C′ should be each equal to O E (Fig. 50*a*), or the pattern will not be true. This gives a means of testing its accuracy.

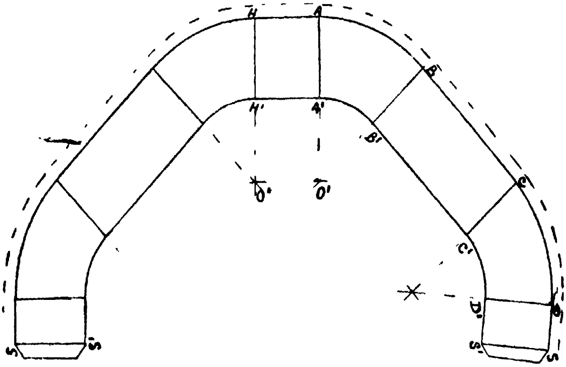
CASE III.—Pattern when the body is to be made up of one piece.

When the body is made in one piece it is usual to have the seam at S (Fig. 49). It is now best to commence with the end pattern.

Draw A A′ (Fig. 52) equal to X′ F (Fig. 49), which, the body being equal-tapering, is the length of its slant anywhere, and draw A H and A′ H′ perpendicular to A A′. Make A H equal to A H (Fig. 49) and draw H H′ perpendicular to A H; then A A′ H′ H will be the end pattern. Next produce A A′, and make A O′ equal to C O (Fig. 50*a*). Then O′ will be the centre for the arcs of the corner pattern, which, drawn by Case II. of this problem, can now be attached at A A′. Through B and B′ draw B C and B′ C′ perpendicular to B B′; make B C equal to B C (Fig. 49), and draw C C′ perpendicular to B C. That completes the addition of the side pattern

B B′ C′ C to the corner pattern. Next produce C C′, mark the necessary centre, and attach to C C′ the corner pattern C D D′ C′, just as A B B′ A′ was attached to A A′. Then through D and D′ draw D S and D′ S′ perpendicular to D D′; make D S equal to D S (Fig. 49) and draw S S′ perpendicular to D S; this adds half an end-pattern to D D′. By a repetition of the foregoing working on the H H′ side of the end-pattern H H′ A′ A, the portion H H′ S′ S on the H H′ side may be drawn, and the one-piece pattern S H A S S′ A′ H′ S′ of the body we are treating of completed.

Fig. 52.



PROBLEM XXVIII.

To draw, **without long radii,** *the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and ends, and round corners; the height and the dimensions of the top and bottom being given.*

This is a fourth case of the preceding problem, and we will apply it to the first case of that problem, that is, when the body is to be made up of four pieces; now, however, assuming that the pieces are so large that their patterns cannot be conveniently drawn by the method there given.

Draw (Fig. 53*a*) P S S′ P′, one-quarter of the plan of the body. Divide C D the larger arc of the plan of the round corner, into three or more equal parts, and join each division-point to O the centre of the arcs of the plan. Draw D F perpendicular to D D′ and equal to the given height of the body, and join D′ F; then D′ F will be the true length of D D′. Next join D 2′; draw D E perpendicular to it and equal to the given height, and join 2′ E; then 2′ E may be taken as the true length of D 2′. (In the fig. a portion of the line 2′ E coincides with the line from 2 to 2′. This is, however, merely a coincidence of the particular case.)

FIG. 53*a*.

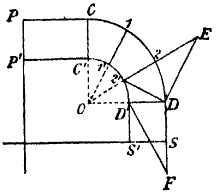
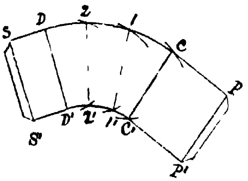


FIG. 53*b*.



Now draw (Fig. 53*b*) D D′ equal to D′ F (Fig. 53*a*), and with D and D′ as centres and radii respectively D 2 and 2′ E (Fig. 53*a*) describe arcs intersecting in point 2. Also with D′ and D as centres and radii respectively D′ 2′ and 2′ E describe arcs intersecting in point 2′. Repeating this construction, but using points 2 and 2′ just found as centres instead of D and D′, the points 1 and 1′ can be found. Similarly using points 1 and 1′ as centres and repeating the construction we can find the points C and C′. Join C C′; draw a regular curve from C through 2 and 1 to D, and another regular curve from D′ through 2′ and 1′ to C′; then in D C C′ D′ we have the pattern of the corner. To this corner pattern, to get at the pattern we require, we have to attach a half-end and a half-side pattern.

Draw D S perpendicular to D D′ and equal to D S (Fig. 53*a*). Also draw S S′ perpendicular to D S, and through D′ draw D′ S′ parallel to D S. The half-end pattern is now attached.

Through C and C′ draw C P and C′ P′ perpendicular to C C′. Make C P equal to C P (Fig. 53*a*), and draw P P′ perpendicular to C P. This attaches the half-side pattern, and completes the pattern required.

If the body is to be made in more than four pieces it will still be generally possible to have a complete corner in one of the pieces. The corner should always be marked out first, and whatever has to be attached should be added as described.

PROBLEM XXIX.

To draw the pattern for an oval equal-tapering body with top and bottom parallel, the height and the dimensions of the top and bottom being given.

Again we deal with four cases; three in this problem and one in the following.

CASE I.—Patterns when the body is to be made up of four pieces.

The plan of the body with lines of construction, drawn by Problem XVIII., is shown in Fig. 54. It will be evident from the plan and from *d*, p. 55, that the body is made up of two round equal-tapering bodies (frusta of cones); the ends C D D′ C′, F E E′ F′, being equal portions of a frustum of one cone, and the sides, E C C′ E′, D F F′ D′, equal portions of a frustum of another cone. Such being the case, it is clear that we require two patterns, one for the two ends, and the other for the two sides; also that the seams should be at E, C, D, and F, where the portions meet.

To draw (Fig. 55*b*) the ends' pattern.

From any point A draw two lines O A, B A (Fig. 55*a*)

FIG. 54.

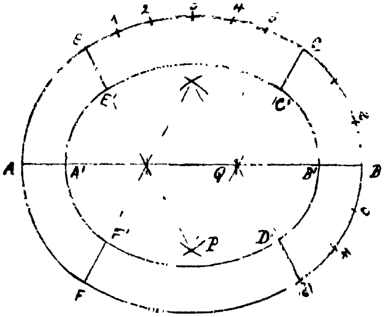


FIG. 55a.

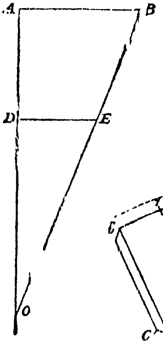


FIG. 55b.

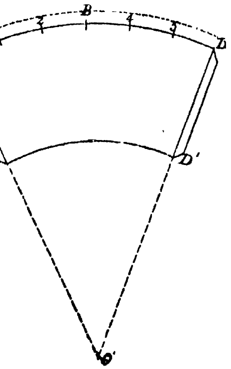


FIG. 56a.

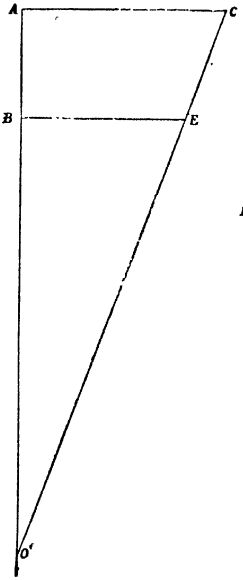
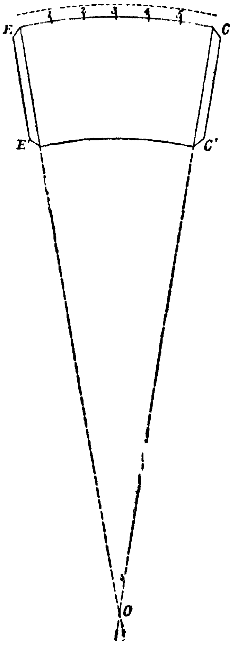


FIG. 56b



perpendicular to each other; make A B equal to Q B (Fig. 54) the radius of the larger end-curve in the plan, and A D equal to the height of the body. From D draw D E perpendicular to A O and equal to Q B′ (Fig. 54) the radius of the smaller end-curve of the plan; join B E and produce it, meeting A O in O. Then with any point O′ (Fig. 55*b*) as centre, and O B (Fig. 55*a*) as radius, describe an arc C D; and with same centre and O E (Fig. 55*a*) as radius describe an arc C′ D′. Join any point C in the arc C D to O′ by a line cutting the arc C′ D′ in C′; divide the arc C D (Fig. 54) into any number of equal parts, and set off from C (Fig. 55*b*) along C D parts equal to and as many as C D (Fig. 54) is divided into. Join D O′ cutting the arc C′ D′ in D′; then D D′ C′ C will be the pattern required.

To draw the pattern for the sides.

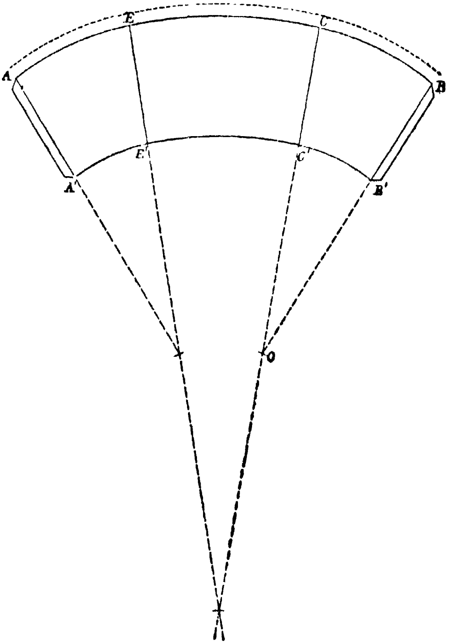
From any point A draw two lines A C, A O perpendicular to each other, and make A C equal to P C (Fig. 54), the radius of the larger side-curve and A B equal to the given height of the body. From B draw B E perpendicular to A B and equal to P C′, (Fig. 54) the radius of the smaller side-curve. Join C E and produce it, meeting A O in O. With any point O′ (Fig. 56*b*) as centre and O C (Fig. 56*a*) as radius describe an arc C E; and with same centre and O E (Fig. 56*a*) as radius describe an arc C′ E′. Now join C O′, and, proceeding exactly similarly as in drawing the ends' pattern, complete E E′ C′ C the pattern for the sides.

CASE II.—Patterns when the body is to be made up of two pieces.

In this case the seams are usually put at the ends A and B (Fig. 54). It is evident that one pattern only is now required, and that this is made up of a side pattern, with, right and left attached to it, a half-end pattern.

Draw (Fig. 57) a side pattern E C C′ E′ in the way described in Case I. Then with C as centre and the radius of the ends' pattern, that is B O (Fig. 55*a*) as radius, set off the distance C O. With O as centre and radii OC and OC′ describe arcs C B and C′ B′. Make C B equal to C B (Fig. 54) and join B O, cutting C′ B′ in B′. Then C B B′ C′ is a half-end pattern attached to C C′. The half-end pattern E A A′ E′ is

FIG. 57.



added at E E′ by repeating the construction just described. This completes A B B′ A′ the pattern required.

Case III.—Pattern when the body is to be made up of one piece.

We will put the seam at A (Fig. 54), the middle of one end. Draw C D D′ C′ (Fig. 58), an end pattern in the way described in Case I. Produce C C′ and make C O equal to

C O (Fig. 56*a*), the radius for a side pattern. With O as centre and radii O C and O C′ describe arcs C E, C′ E′. Make

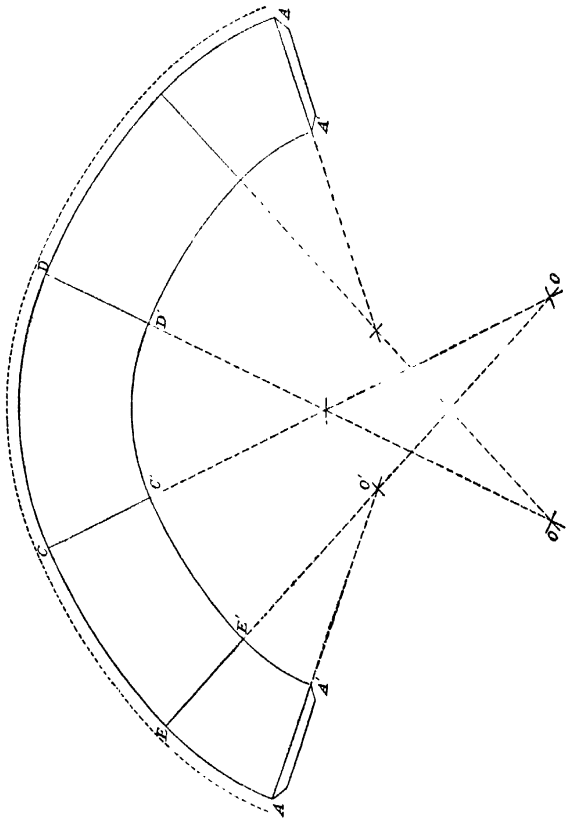


FIG. 58.

C E equal to C E (Fig. 54) and join E O cutting C′ E′ in E′. Make E O′ equal to B O (Fig. 55*a*) and with O′ E as radius describe an arc E A. With same centre and O′ E′ as radius describe an arc E′ A′. Make E A equal to E A (Fig. 54) and join A O′, cutting E′ A′ in A′. The remainder D A A′ D′ of the pattern can be drawn by repeating the foregoing construction. The figure A C D A A′ D′ C′ A′ will be the pattern required.

PROBLEM XXX.

To draw, **without long radii,** *the pattern for an oval equal-tapering body with top and bottom parallel, the height and the dimensions of the top and bottom being given.*

This is a problem which will be found very useful for large work, especially with the pattern for the sides, the radius for which is often of most inconvenient length.

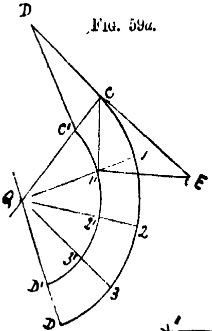


FIG. 59a.

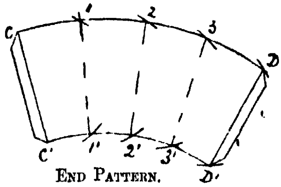


FIG. 59b

END PATTERN.

To draw the ends' pattern.

Draw (Fig. 59a) the plan C D D′ C′ of the end of the body. Divide C D into four or more equal parts, and join, 1, 2, &c.,

to Q, by lines cutting C′ D′ in 1′, 2′, &c. From C draw C D perpendicular to C C′ and equal to the given height. Join C′ D, then C′ D is the true length of C C′. Join C 1′; draw 1′ E perpendicular to it and equal to the given height; and join C E. Then C E may be taken as the true length of C 1′. Draw C C′ (Fig. 59*b*) equal to C′ D (Fig. 59*a*). With C and C′ as centres and radii respectively equal to C 1 and C E (Fig. 59*a*) describe arcs intersecting in point 1. With C′ and C as centres and radii respectively equal to C′ 1′ and C E (Fig. 59*a*) describe arcs intersecting in point 1′. By using points 1 and 1′ as centres and repeating the construction, points 2 and 2′ can be found. Similarly find points 3 and 3′, and D and D′. Join D D′, draw a regular curve from C through 1, 2, and 3, to D, and another regular curve from C′ through 1′, 2′, and 3′, to D′. The figure C C′ D′ D is the pattern required.

FIG. 60*a*.

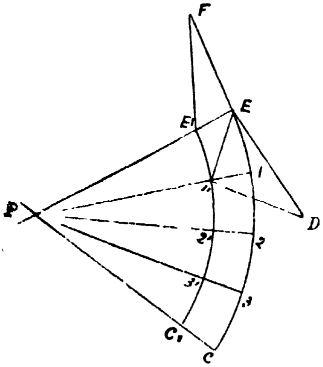
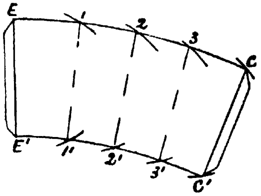


FIG. 60*b*.



SIDE PATTERN.

To draw the sides' pattern.

Draw (Fig. 60*a*) the plan E C C′ E′ of the side of the body. Divide E C into four or more equal parts, and join 1, 2, &c., to P, by lines cutting E′ C′ in 1′, 2′, &c. From E draw E F perpendicular to E E′ and equal to the given height. Join E′ F; then E′ F is the true length of E E′. Next join E 1′; draw 1′ D perpendicular to E 1′ and equal to the given height, and join E D; then E D may be taken as the true length of E 1′. Now draw (Fig. 60*b*) E E′ equal to E′ F (Fig. 60*a*), and with E and E′ as centres and radii respectively equal to E 1 and D E (Fig. 60*a*) describe arcs intersecting in point 1. With E′ and E as centres and radii respectively equal to E′ 1′ and D E (Fig. 60*a*) describe arcs intersecting in point 1′. Repeating the construction with points 1 and 1′ as centres, the points 2 and 2′ can be found; and similarly the points 3 and 3′ and the points C and C′. Join C C′; draw a regular curve from E through 1, 2, and 3, to C, and another from E′ through 1′, 2′, and 3′ to C′. The figure E E′ C′ C will be the pattern required

Book III.

CLASS III.

Patterns for Miscellaneous Articles.

The book we now reach is made up of miscellaneous problems, all of which are of considerable practical importance, as well as are also typical cases.

PROBLEM I.

To draw a pattern for the elbow formed by two equal circular pipes (cylinders of equal diameters) which meet at any angle.

First draw (Fig. 1) a side elevation and a part-plan of the elbow, as follows. Draw any two lines G G′, O G′ at an angle to each other equal to the given angle of the elbow; through any point G in G G′ draw G A perpendicular to G G′ and equal to the diameter of the pipes; and through A draw an indefinite line A A′ parallel to G G′. Then from any point O in G′O draw O N perpendicular to G′O and equal to the diameter of the pipe; and through N draw an indefinite line N A parallel to G′O and meeting A A′ in A′. Join A′G′ (the line A′G′ produced always bisects the angle G G′O); then O N A G is a side elevation of the elbow, and the line A′G′ represents the joint or 'mitre' of the pipes. On G A describe a semicircle G *d* A (this will be the part-plan of the elbow); which divide into any number, here six, of equal parts in the points *b*, *c*, *d*, *e*, *f*. From each of these points draw lines parallel to A A′, namely, the lines *b* B′, *c* C′, *d* D′, *e* E′, and *f* F′, cutting G A in the points B, C, D, E, and

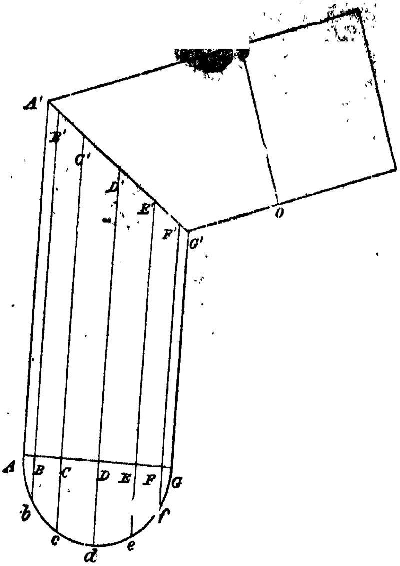
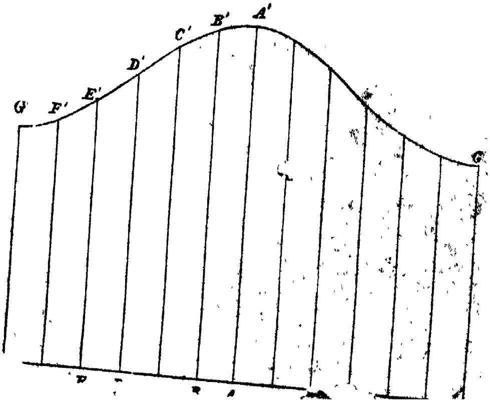


FIG. 2.



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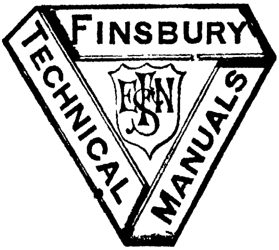

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TO

THE THIRD EDITION.

THE *Third Edition* of this work contains, with 50 new diagrams and illustrations, more than 80 pages of additional matter. Books I. and II. remain as before. Book III. contains several new and important problems in "penetrations." Book IV. is almost new. Towards Chapter I., which describes Pipe-Bending, help was furnished to the author by his former student and assistant, Mr. W. H. Bennett, now Instructor in *Metal-Plate Work* at the Regent Street Polytechnic. Chapter II. treats of the making up of Pipe Bends out of flat sheet, and deals with that subject as it has never before been dealt with—systematically and comprehensively. To Book V. has been added an article on "Tinning and Re-tinning."

The system of geometric construction for the patterns of sheet-metal articles set forth in this book, came of investigation made by the author about twenty-two years ago. In 1885, after several years of thorough testing, the system was illustrated in model and diagram at the South Kensington Inventions Exhibition, and in 1887 further made public in the First Edition. A workman, studying the book, learns to recognise easily the nature of any surface he meets with, whether flat or curved, and how to deal with it for his purposes. Indirect evidence of the value of the book is afforded by the extent to which other writers—with no acknowledgement, however, of their indebtedness—have turned to it for "copy" and model; and direct evidence in the frequent communications received from actual sheet-metal workers, who, by the author's system, have readily solved problems, very difficult if not impossible of solution otherwise.

As with the Second Edition, so now, I have to acknowledge the very valuable assistance given me by my friend, Mr. H. W. Ley, in revising the new matter, and seeing the book through the press.

C. T. M.

BOROUGH POLYTECHNIC INSTITUTE, LONDON.
April, 1899.

PREFACE

TO

THE SECOND EDITION.

THIS *Second Edition*, long delayed through pressure of work, is the First Edition augmented by about 70 more pages of original matter, making the book still more useful to the sheet-metal worker who desires to know his trade, and become an all-round hand, instead of being cramped up in a narrow everyday groove. The specialisation in the shops brought Technical Education about; the increasing tendency to it marks the need of a wide basis for such education.

The divisions of tin-plate, iron-plate, and zinc workers did not formerly exist; they do not exist now among the older workmen, and should not among the younger. The shapes treated in this volume come before all sheet-metal workers; the main differences of their work are not those of shape, but of size and consequent manipulation.

Blinn's Workshop Companion for Tin, Sheet Iron, and Copper Plate Workers,' is a book published in America and circulating here. More than one-quarter of that portion which deals with patterns in its 1891 edition is neither more nor less than a word for word appropriation, without acknowledgment, of original problems, diagrams and all, from my pages. The *Preface* to the book can well afford to make much of "the addition of new matter."

My thanks are due to Mr. H. W. Ley, Secretary of the Westminster Technical Institute during my Directorship of it, for many useful suggestions, and for very careful revision of its proof sheets.

C. T. M.

PREFACE.

In the pages that follow, the setting-out of patterns for metal-plate workers is *systematised* (for the first time, so far as I am aware), and by the system laid down, nearly all the patterns required by sheet-metal workers can be set out on one general geometrical principle. Thus treated, the subject becomes a comparatively easy one, and the workman learns how, on the given principle, to develop for himself the surface of any article he may have to make, to the saving, as against various methods practised hitherto, of both time and material.

Of the methods heretofore taught in the workshop or otherwise, some have no application beyond the particular article with respect to which they are described, some are absolutely impracticable. The methods that these pages explain are applicable always; and have been proved by abundant experience in my classes.

The commencement of the book was a series of articles that were written by me for *The Ironmonger.* These and my class and lecture notes have greatly aided me in my effort to make the book comprehensive, as well as at the same time a welcome workshop companion. Not only does it contain patterns which are essentially those of numerous articles of every-day manufacture, but a *special feature* is made of large work;—and how it may be dealt with under ordinary workshop conditions. Each of the problems is complete in itself; but although solved independently they follow each other in due order.

It is my hope that the book will be an aid to students in engineering and geometry, as well as to those for whom it is particularly written; that it will be a serviceable addition to the scanty and insufficient literature on the application of geometry to metal-plate work; and that it will in some degree assist the cause of technical education.

C. T. M.

BOOK II.

CHAPTER I.

PATTERNS FOR ROUND ARTICLES OF UNEQUAL TAPER OR INCLINATION OF SLANT.

(CLASS II. *Subdivision a.*)

(42.) WE stated at the commencement of the preceding division of our subject that it was advisable, in order that the rules for the setting out of patterns for articles having equal slant or taper should be better understood and remembered, to consider the principles on which the rules were based; and the remark is equally true and of greater importance in respect of the rules for the setting out of patterns for articles having unequal slant or taper. We have shown that the basis of the rules for the setting out of patterns for equal tapering bodies is the right cone; we purpose showing that the basis of the rules for the setting out of patterns for articles of unequal slant or taper is what is called the *oblique* cone. The consideration of the cone apart from its species, that is, apart from whether it is right or oblique, which becomes now necessary, immediately follows.

DEFINITION.

(43.) *Cone.*—A cone is a solid of which one extremity, the base, is a circle, and the other extremity is a point, the apex. The line joining the apex and the centre of the base is the axis of the cone.

(44.) Given a circle and a point *in* the line passing through the centre of the circle at right angles to its plane. If an indefinite straight line, passing always through the given point, move through the circumference of the given circle, there will be thereby generated between the point and the circle, a solid; this solid is the *right* cone.

(45.) If, all other conditions remaining the same, the given point is *out of* the line passing through the centre of the circle at right angles to its plane, the solid then generated will be the *oblique* cone.

(46.) Figs. 1 and 2 represent oblique cones. The lines VA, VC, VF, and VG (either fig.) drawn from the apex V to the circumference of the base of the cone, are portions of the generating line at successive stages of its revolution. It is but a step from this and will be a convenience, to regard these lines, first, as each of them part of an independent generating line, and then as each of them a complete generating line. Shaded representations of various oblique cones will be found later on.

FIG. 1.

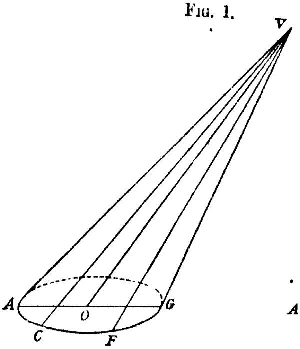
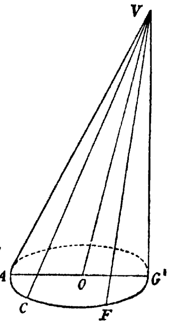


FIG. 2.



(47.) Comparing now the right and oblique cones. A right cone may be said to be made up of an infinite number of equal generating lines, and an oblique cone of an infinite number of unequal generating lines.

(48.) If a right cone is placed on a horizontal plane, the apex is vertically over the centre of the base.

(49.) If when a cone is placed on a horizontal plane the apex is not vertically over the centre of the base, the cone is oblique. Hence all cones not right cones are oblique cones. Hence also a cone is oblique if its axis is not at right angles to every diameter of its base.

(50.) In the right cone, any plane containing the axis is perpendicular to the base of the cone, and contains two generating lines (see Figs. 1*a* and 2*a*, p. 25).

(51.) In the oblique cone, only *one plane* containing the axis is perpendicular to the base of the cone, and this plane contains its longest and shortest generating lines. In Fig. 1 or 2 the lines V A and V G are respectively the longest and shortest generating lines, and the plane that contains these lines contains also V O the axis of the cone.

(52.) The obliquity of a cone is measured by means of the angle that its axis makes with that radius of the base that terminates in the extremity of the shortest generating line. Thus the angle V O G gives the obliquity of either cone V A G. The angle V O G also gives the inclination of the axis. As the angle V A G is in the same plane as V O G and smaller than that angle, it will be seen that not only is V A the longest generating line, but it is also the line of greatest inclination on the cone. Similarly as the angle that V G makes with A G produced is greater than V O G, the line V G is not only the shortest generating line, but also the line of least inclination on the cone.

(53.) The plane that contains the longest and shortest generating lines bisects the cone; consequently the generating lines of either half are, pair for pair, equal to one another.

(54.) If the elevation (see § 21, p. 48) of an oblique cone be drawn on a plane parallel to the bisecting plane, the elevations of the longest and shortest generating lines will be of the same lengths respectively as those lines. Thus if the triangle V A G (either fig.) be regarded as the elevation of the cone represented, then V A will be the true length of the longest generating line of the cone, and V G of the shortest. In speaking in the pages that follow, of elevation with regard to the oblique cone, we shall always suppose it to be on a plane parallel to the bisecting plane.

(55.) If the hypotenuse of a right-angled triangle represent a generating line of any cone, right or oblique, then one of the sides containing the right angle is equal in length to the plan of that line, and the other side is equal to the height (distance between the extremities) of any elevation of it. See, p. 25, the triangles O B A, O G A, O E A, O D C, O K C, O F C, and, p. 109, the triangles V' A V, V' B V, V' C V, &c. See also § 22*a*.

(56.) As the generating lines of the oblique cone vary in length, the setting out of patterns of articles whose basis is the oblique cone (that is to say, the development of the curved surfaces of such articles) differs from that appertaining to articles in which the right cone is involved. The principles, however, are the same in both cases. In developments of the right cone, if a number of its generating lines are laid out in one plane, then as they are all equal and have a point (the apex) in common, the curve joining their extremities is an arc of a circle of radius equal to a generating line; while in developments of the oblique cone, as the generating lines, though they still have a point in common, vary in length, the individual lengths of a number of them must be found, as well as the distances apart of their extremities, before, by being laid out in one plane at their proper distances apart, a curve can be drawn through their extremities, and the required pattern ascertained.

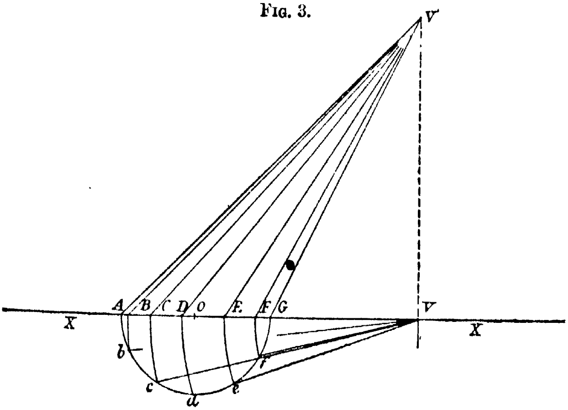
The problem which follows gives the working of this in full.

PROBLEM I.

To draw the pattern (develop the surface) of an oblique cone, the inclination of the axis (§ 52), its length and the diameter of the base of the cone being given.

First (Fig. 3) draw V' A G the elevation of the cone, and A *d* G half the plan of its base; thus. Draw any line X X, and at any point O in it, make the angle V' O X equal to the given inclination of the axis (a line O V' is omitted in the fig. to avoid confusion), and make O V' equal to the given length of the axis. With O as centre and half the given diameter of the base as radius describe a semicircle A d G cutting X X in A and G. Join A V′, G V′, then V′ A G is the elevation required, and A d G is the half-plan of base. Next divide A d G into any convenient number of parts, equal or unequal. The division here is into six, and the parts are equal; to make them so being an advantage. Now let fall V′ V perpendicular to X X; and join V to the division points b, c, d, e, f (to save multiplicity of lines this is only partially shown in the fig.). The lines V A, V b, V c, &c., will be the lengths in plan of seven lines from apex

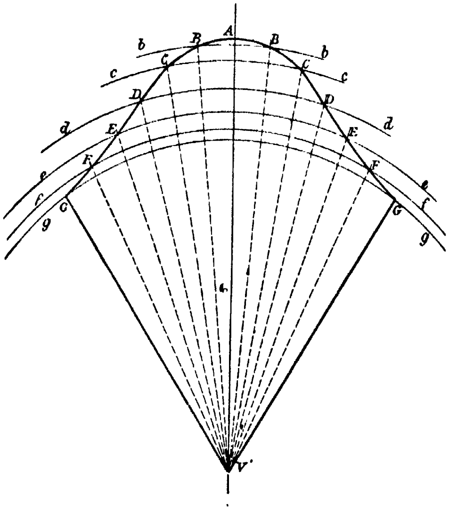
FIG. 3.



to base of cone, that is, of seven generating lines. Now with V as centre and radii successively V b, V c, V d, V e, and V f, describe arcs respectively cutting X X in B, C. D, E, and F. Join V′ B, V′ C, &c.; then as V′ V is the height of any elevation of either of the seven generating lines (see § 22a, p. 48, and § 55, p. 107) and as V A, V B, &c., are their plan lengths, we have in V′ A, V′ B, V′ C, &c., their true lengths. V′ G is not only so however, it is (§ 54) the shortest of all the generating lines of the cone.

To draw the pattern of the cone with the seam to correspond with V′ G the shortest generating line. Draw (Fig. 4) V′ A equal to V′ A (Fig. 3), and with V′ as centre and V′ B, V′ C, V′ D, V′ E, V′ F, and V′ G (Fig. 3) successively as radii describe arcs, respectively, *b b*, *c c*, *d d*, *e e*, *f f*, and *g g*. With A as centre and radius equal to A *b* (A *b* being the distance apart on the round of the cone at its base of the generating lines V′ A and V′ B′) (Fig. 3) describe an arc cutting the arc *b b* right and left of V′ A in B and B. With these points B and B as centres and radius as before (the distances apart of the extremities of the generating lines at the base of the cone being all equal), describe arcs cutting the arc *c c* right and left of V′ A in C and C. With same radius and the last-named points as centres describe arcs cutting *d d* right and left of V′ A in D and D. With D and D as centres and same radius describe arcs cutting *e e* right and left of V′ A in E and E, and with E and E as

Fig. 4.



centres and same radius describe arcs cutting *ff* right and left of V′ A in F and F. Similarly with same radius and F and F as centres find points G and G. Draw through A and the points B, C, D, E, F and G, right and left of V′ A an unbroken curved line. Also join G V′ right and left of V′ A, then G A G V′ will be the pattern required.

The dotted lines V′ B, V′ C, V′ D, &c., are not necessary for the solution of the problem, and are only drawn to show the position of the seven generating lines on the developed surface.

(57.) As the plane of the elevation V′ A G bisects the cone (§§ 53 and 54), it is clear that the seven generating lines found, correspond with seven other generating lines on the other half of the cone; so that in finding them for one half of the cone, we find them for the other, as the halves necessarily develop alike.

(58.) Round articles of unequal slant or taper having their ends parallel are portions (frusta) of oblique cones.

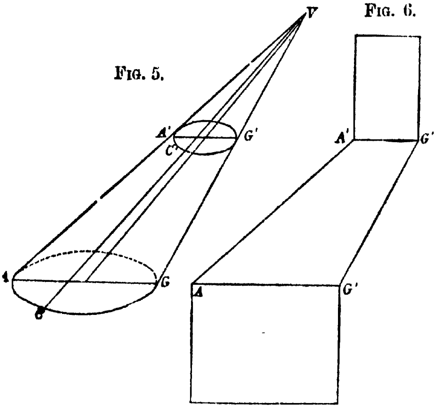


Fig. 5.

Fig. 6.

Definition.

(59.) *Frustum.*—If an oblique cone be cut by a plane parallel to its base, then the part containing the apex is still an oblique cone as V A′ C′ G′ (Fig. 5), and the part A′ A C G G′ containing the base is a *frustum* of an oblique cone. A shaded representation of such frustum will be found in Fig. 15. This frustum, however, differs from that of the right cone (§ 12, p. 33), in that, though having circular ends, its sides are not of equal, but of unequal slant. Conversely, a round unequally tapering body, having its top and base parallel, is a frustum of an oblique cone. A tapering piece of pipe A′ A G G′ (Fig. 6) joining two cylindrical pieces which are not in line with each other is a frustum of an oblique cone.

(60.) Referring to Fig. 5, it is evident that if the pattern for the larger cone, V A G, and the pattern for the smaller cone, V A′ G′, be drawn from a common centre V′ (Fig. 9), the figure G′ G A G G′ (Fig. 9) will be the pattern for the portion A G G′ A′ (Fig. 5) of the cone V A G. The line C C′ shows a generating line of the frustum (*b*, p. 126).

FIG. 7*a*.

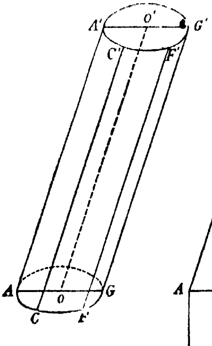
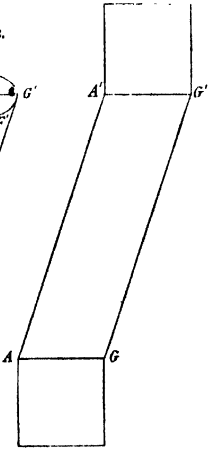


FIG. 7*b*.



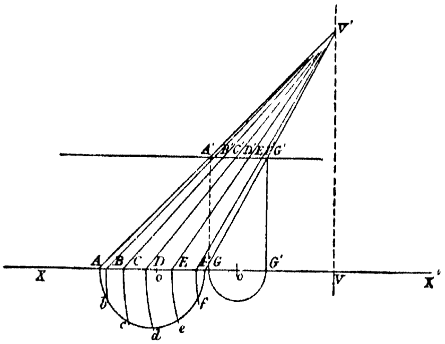
(61.) A case that not unfrequently occurs needs mention here. Suppose the diameters of the top and base of a round unequal-tapering body of parallel ends are very nearly equal, then it is evident that the apex of the oblique cone of which the body is a portion will be a long distance off, and, if the diameters be equal, then the body becomes what is called an oblique cylinder, (a cylinder is a round body without any taper at all). Of course this is an extreme case, but it is quite an admissible one. For, if the diameters of the ends of the body differ by only $\frac{1}{10000}$ inch, then, clearly, however short the body may be, we are dealing with a frustum of an oblique cone, although so nearly a cylinder that, for almost any purpose occurring in practice, it could be treated as a cylinder. Later on the advantages of looking upon the oblique cylinder as a special case of frustum of an oblique cone, and considering its generating lines as parallel, will be seen. Such a frustum is represented in A A′ G′ G in Figs. 7*a* and 7*b*, the line O O′ being the axis of the frustum and C C′ and F F′ generating lines. A construction easily dealing with its development will be given presently.

PROBLEM II.

To draw the pattern of a round unequal-tapering body with top and base parallel (frustum of an obliquecone, as in Fig. 5), the diameters of top and base, the height, and the inclination of the longest generating line being given.

First (Fig. 8) draw the elevation and half the plan of the body, thus: Draw a line X X′, and at any point A in it, make the angle X′ A A′ equal to the given inclination of the longest generating line. At a distance from X X′ equal to the given height, draw a line A′ G′ parallel to X X′. From the point A′ where A′ G′ cuts A A′, make A′ G′ equal to the diameter of the top, also make A G equal to the diameter of the base; and join G G′. Then A A′ G′ G is the elevation of the body. Now on A G describe a semicircle A *d* G, this will be half the plan of the base. Produce A A′, G G′, to intersect in V′; this point will be the elevation of the apex of the cone of which the body is a portion. Through V′ draw V′ V perpendicular to X X′; divide the semicircle into any convenient number of equal parts, here six, in the

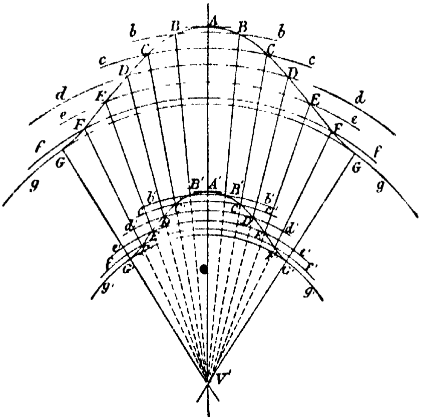
Fig. 8.



points *b*, *c*, *d*, *e*, and *f*; with V as centre and radii successively V *b*, V *c*, V *d*, &c., describe arcs respectively cutting X X in B, C, D, &c.; and join B, C, D, &c., to V′ by lines cutting A′ G′ in B′, C′, D′, E′, and F′. Then B B′, C C′, D D′, &c., will be (see construction of last problem) the true lengths of various generating lines of the frustum (*b*, p. 126).

Now to draw the pattern (Fig. 9) so that the seam shall correspond with G G′ (Fig. 8) the shortest generating line. Draw (Fig. 9) V′ A equal to V′ A (Fig. 8) and with V′ (Fig. 9) as centre and radii successively equal to V′ B, V′ C, V′ D, V′ E, V′ F, and V′ G (Fig. 8), describe, respectively, arcs *b b*, *c c*, *d d*, *e e*, *f f*, and *g g*. With A as centre and radius equal A *b* (Fig. 8) (see preceding problem for the reason of this), describe arcs cutting the arc *b b* right and left of V′ A in B and B. With these points, B and B as centres and radius as before, describe arcs cutting the arc *c c* right and left of V′ A in C and C. With same radius and the last named points as centres, describe arcs cutting *d d* right and left of V′ A in D and D. With D and D as centres and same radius, describe arcs cutting *e e* right and left of V′ A in E and E, and with E and E as centres and same radius describe arcs cutting *f f* in F and F. Similarly, with same radius and F and F as centres find points G and G. Join the points B, C, D, E, &c., right and left of V′ A to V′. With V′ as centre and V′ A′ (Fig. 8) as radius describe an arc

Fig. 9.



cutting V′ A in A′. With same centre and V′ B′ (Fig. 8) as radius describe an arc *b′ b′* cutting V′ B right and left of V′ A in B′. With same centre and V′ C′ (Fig. 8) as radius describe an arc *c′ c′* cutting V′ C right and left of V′ A in C′. Similarly, with the same centre and V′ D′, V′ E′, V′ F′, and V′ G′ (Fig. 8) successively as radii describe, respectively, arcs *d′ d′*, *e′ e′*, *f′ f′*, and *g′ g′*, cutting V′ D, V′ E, V′ F, and V′ G, right and left of V′ A respectively in D′, E′, F′, and G′. Draw through A and the points B, C, D, E, F, and G, right and left of V' A an unbroken curved line. Also draw through A' and the points B', C', D', E', F', and G', right and left of V' A an unbroken curved line. Then G A G G' A' G' will be the pattern required.

The small semicircle (Fig. 8) and perpendicular lines from its extremities are not needed in this problem, but are introduced in illustration of that next following.

(62.) In the applications of the oblique cone, it is generally in the form next following that the problem presents itself.

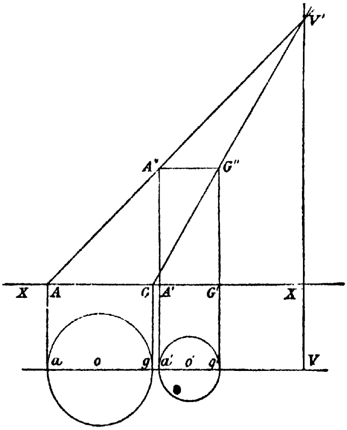
PROBLEM III.

To draw the pattern of a round unequal-tapering body with top and base parallel (frustum of oblique cone), its plan and the perpendicular distance between the top and base (the height of the body) being given.

The working of this problem should be carefully noted, for the reason just above stated.

Let $a\,g\,a'\,g'$ (Fig. 10) be the given plan of the body. The side lines of the plan are not drawn, but only the circles of its top and base, as we do not make use of the side lines. In fact, all that we really make use of is, as will be presently seen, the halves of the plan-circles of the top and base of the body, not, however, placed anyhow, but *in their proper positions relatively to one another in plan.* This is the all-essential point. If the plan-circle $a'\,g'$ (Fig. 10) of the top of the frustum were further removed than represented from the plan-circle $a\,g$ of its base, we should, by the end of our working, get at the pattern of some other oblique-cone frustum than that the pattern of which we require. Through O O' the centres of the circles draw an indefinite line a V, and draw any line X X parallel to O O. Through a and g draw a A and g G perpendicular to X X, and through a' and g' draw indefinite lines a' A'' and g' G'' perpendicular to X X, and cutting it in points A′, G′. Make A′ A″ and G′ G″ each equal to the given height or perpendicular distance that the top and base are apart. Join A A″ and G G″, and produce these lines to intersect in V′. The problem can now be completed by Problem II.

Fig. 10.



It should be noted that it is only necessary to draw half the plans of the top and base as shown by the semicircles A G and A′ G′ in Fig. 8.

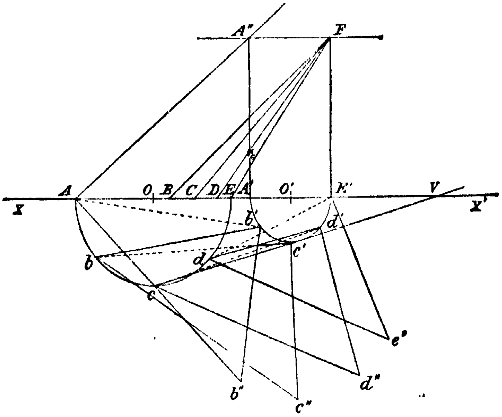
The case of large work, where long radii would be inconvenient, we treat as a separate problem, and we will suppose the dimensions given to be those of Problem II. The method is also suitable where there is little difference between the diameters of top and base of the body.

PROBLEM IV.

To draw **without long radii** *the pattern of a round unequal-tapering body with top and base parallel, the diameters of the top and base, the height, and the inclination of the longest generating line being given.*

Draw any line X X′ (Fig. 11) and at point A in it make the angle X′ A A″ equal to the given inclination of the longest generating line. At a distance from X X′ equal to the given height draw a line A″ F parallel to X X′. Make A E equal to the longer of the given diameters, and on it describe the semicircle A *c* E. From the point A″ where A″ F cuts A A″ let fall A″ A′ perpendicular to X X. Make A′ E′ equal to the shorter of the given diameters, and on it describe the semicircle A′ *c*′ E′. Next find *c* and *c*′ the mid-points of the semicircular arcs. These are easily found by drawing lines (not shown in the fig.) through O and O′ (the

Fig. 11.



centres from which the semicircles are described) perpendicular to X X′ and cutting the semicircular arcs in the required points *c* and *c*′. Join *c c*′ and produce it meeting X X′ in V; this point will be the plan of the apex of the cone of which our tapering body is a frustum. This point is by no means always within what may be termed workable reach, as for instance where the two semicircles are nearly equal the lines X X′ and *c c*′ therefore very nearly parallel, and the producing *c c*′ to V impracticable. We will work under both suppositions.

(63.) If V is accessible, then divide the larger semicircle into any convenient number of parts (four parts only are taken in the figure in order to keep it clear), as A *b*, *b c*, *c d*, *d* E. Join *b* and *d* to V (*c* is already thus joined, the lines from *b* and *d* to V are only drawn in the fig. as far as the smaller semicircle) by lines cutting the smaller semicircle in *b*′ and *d*′. Then A A′, *b b*′, *c c*′, &c., are the plans of generating lines of the frustum (tapering body), and in order to draw its pattern their true lengths must be found.

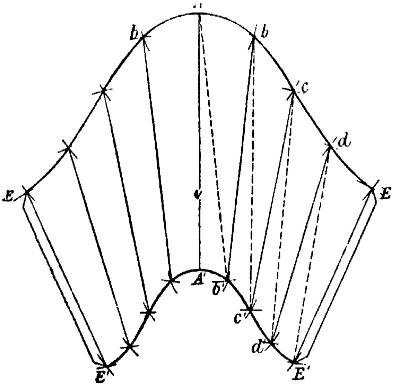
(64.) If V is inaccessible, then divide the smaller semicircle into four parts (the same 'convenient number' of parts that the larger semicircle was divided into), in the points *b*′, *c*′, and *d*′, and join *b b*′, *c c*′, and *d d*′.

The true lengths of *b b*′, *c c*′, &c., are found as follows: From E′ draw a line perpendicular to X X′ and cutting A″ F in F, and join E F. Then E F is the true length of E E′. Now make E′ D equal to *d d*′ and join D F; then D F is the true length of *d d*′. Next set off E′ C equal to *c c*′, and E′ B equal to *b b*′; and join C F and B F. Then C F and B F are the true lengths respectively of *c c*′ and *b b*′. The true length of A A′ we already have in A A″, and as this is the longest generating line of the frustum, E F will be the shortest.

We proceed now to find the distance the points A and *b*′, *b* and *c*′, *c* and *d*′, &c, are apart, which we do by finding the true lengths of the lines A *b*′, *b c*′, *c d*′, and *d* E′, joining the points. Through *b*′ draw *b*′ *b*″ perpendicular to A *b*′, and equal to the given height. Join A *b*″; then A *b*″ may be taken as the true length of A b'. Similarly, through c', d', and E′ draw lines equal to the given height, and perpendicular to $b\,c'$, $c\,d'$, and d E′ respectively. Join $b\,c''$, $c\,d''$, and $d\,e''$; then $b\,c''$, $c\,d''$ and $d\,e''$ may be taken as the true lengths required.'

To draw the pattern (Fig. 12) the seam to correspond with the shortest generating line. Draw A A′ equal to A A″ (Fig. 11) and with A and A′ as centres and radii respectively A b'' and A′ b' (Fig. 11) describe arcs intersecting in b'. Next with b' and A as centres and radii respectively B F and A b (Fig. 11) describe arcs intersecting in b. Then A, b, A′, b', are points in the curves of the pattern. With b and b' as

FIG. 12.



centres and radii respectively $b\,c''$ and $b'\,c'$ (Fig. 11) describe arcs intersecting in c'. With c' and b as centres and radii respectively C F and $b\,c$ (Fig. 11) describe arcs intersecting in c; and with c and c' as centres and radii respectively $c\,d''$ and $c'\,d'$ (Fig. 11) describe arcs intersecting in d'. With d' and c as centres and radii respectively D F and $c\,d$ (Fig. 11) describe arcs intersecting in d. Similarly find E′ and E. Draw unbroken curved lines through A $b\,c\,d$ E and A′ $b'\,c'\,d'$ E and join E E′; that will give us half the pattern. By like procedure we find the other half of the pattern, that to the left of A A′.

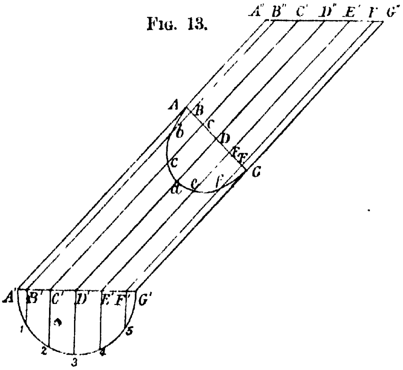
(65.) The lines $b\,b'$, $c\,c'$, &c., and the dotted lines A b', $b\,c'$, &c., are drawn in Fig. 12 simply to show the position that the lines which correspond to them in Fig. 11 ($b\,b'$, A b', &c.) take upon the developed surface of the tapering body. It is evident that it is not a necessity to make distinct operations of the two halves of the pattern; for as the points b', b, c', c, &c., are successively found, the points on the left of A A′ corresponding to them can be set off.

PROBLEM V.

To draw the pattern of an oblique cylinder (inclined circular pipe for example), the length and inclination of the axis and the diameter being given.

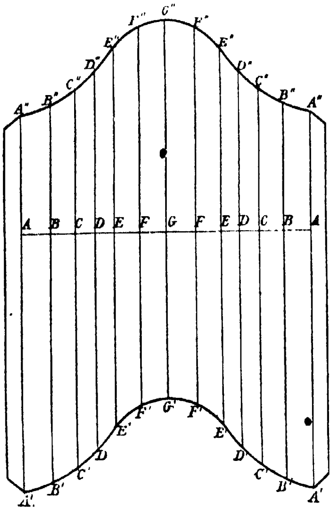
Draw (Fig. 13) any line A′ G′, and at the point A′ in it

FIG. 13.



make the angle G′ A′ A″ equal to the given inclination of the axis. Make A′ G′ equal to the given diameter, and draw a line G′ G″ parallel to A′ A″. Make A′ A″ and G′ G″ each equal to the length of the cylinder (the length of a cylinder is the length of its axis) and join $A''G''$. Then $A'A''G'G''$ is the elevation of the cylinder. Now on $A'G'$ describe a semicircle, and divide it into any number of equal parts, in the points 1, 2, 3, &c.; through each point draw lines perpendicular to $A'G'$, meeting it in points B', C', D', &c., and through B', C', D', &c., draw lines parallel to $A'A''$. Draw any line AG perpendicular to $A'A''$ and $G'G''$, cutting the lines $B'B''$, $C'C''$, &c., in points B, C, &c. Next make Bb equal to $B'1$, Cc equal to $C'2$, Dd equal to $D'3$, Ee equal to $E'4$, and Ff equal to $F'5$, and draw a curve from A through the points b, c, d, &c., to G. It is necessary to remark that this curve is not a semicircle, but a semi-ellipse (half an ellipse).

Fig. 14.



To draw the pattern (Fig. 14). Draw any line AA, and at about its centre draw any line $G''G'$ perpendicular to it and cutting it in G. From G, right and left of it, on the line A A mark distances G F, F E, E D, D C, C B, and B A equal respectively to the distances *G f*, *f e*, *e d*, *d c*, &c. (Fig. 13). Through the points F, E, D, &c., right and left of G, draw lines parallel to G″ G′. Make G G′, G G″ equal to G G′, G G″ (Fig. 13) respectively. Similarly make F F′, F F″, E E′, E E″, D D′, D D″, &c., right and left of G′ G″ equal respectively to F F′, F F″, E E′, E E″, D D′, D D″ &c. (Fig. 13). Draw an unbroken curved line from G″ through F″, E″, D″, &c., right and left of G″ and an unbroken curved line through F′, E′, D′, &c., right and left of G′. The figure A″ G″ A″ A′ G′ A′ will be the pattern required. The two parts G″ A″ A′ G′ of the pattern are alike in every respect.

CHAPTER II.

UNEQUAL-TAPERING BODIES, OF WHICH TOP AND BASE ARE PARALLEL, AND THEIR PLANS.

(66.) BEFORE going into problems showing how to draw the patterns of unequal-tapering bodies with parallel ends, bodies which are (as the student will realise as he proceeds) partly or wholly portions of oblique cones, it will be necessary to enter into considerations in respect of the plans of frusta of such cones (see § 58, p. 111), similar to those appertaining to the plans of frusta of right cones treated of in Chap. V., Book I.; but to us of greater importance, because the constructions in problems for the setting out of patterns of bodies having unequal taper or inclination of slant are a little more difficult than those in problems for patterns of equal-tapering bodies. The chapter referred to may be now again read with advantage.

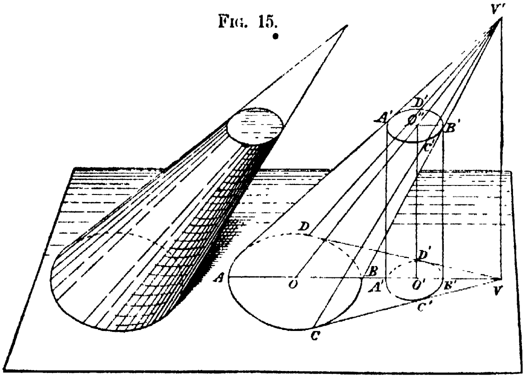
As much use will be hereafter made of the terms *Proportionate Arcs* and *Similar Arcs* we now define them. We also extend our explanation of *Corresponding Points.*

DEFINITIONS.

(67.) PROPORTIONAL ARCS: SIMILAR ARCS.—Arcs are *proportional* when they are equal portions of the circumferences of the circles of which they are respectively parts; they are *similar* when they are contained between the same generating lines. Similar arcs are necessarily proportional. In Fig. 15 the arcs A D and A′ D′ are proportional because each is a quarter of the circumference of the circle to which it belongs. They are similar because the generating lines V′ A and V′ D contain them both.

(68.) CORRESPONDING POINTS.—Points on the same generating line are *corresponding points* (compare § 24); thus, the points A and A′ are corresponding points, because they are on the same generating line V′ A ; also the points C and C′ on the generating line V′ C. The point A′ on V A is the plan of A on V′ A ; the point C′ on V C is the plan of C′ on V′ C ; and so on.

Fig. 15.



(69.) From the figure, which shows a frustum of an oblique cone standing on a horizontal plane, it will be seen that the plan of a round unequal-tapering body (frustum of oblique cone) consists mainly of two circles C A D B, C′ A′ D′ B′, the plans of the ends of the body. In Fig. 16 is shown a complete plan of an oblique-cone frustum. With the connecting lines of the two sides we are not concerned, but may simply mention that they are *tangents* (lines which touch but do not cut) to the circles. Further from Fig. 15 it will be seen that, completing the cone of which the tapering body is a portion.

a. The plan of the axis (line joining the centres of the ends) of a frustum is a line joining the centres of the circles which are the plans of its ends; thus, the line O O′ is the plan of the axis O O″ (see also O O′, Fig. 16).

Similarly, the plan of the axis of the complete cone, is the plan, produced, of the axis of its frustum; thus, O V is the plan of O V'.

b. The plan, produced both ways, of the axis of a frustum contains the plans of the lines of greatest and least inclination on the frustum (see § 52); that is to say, of the longest and shortest lines on it. Thus, O O', produced both ways, contains the plans of A A' and B B'. It is convenient to regard lines joining corresponding points of a frustum (corresponding points of a frustum are points on one and the same generating line of the complete cone) as generating lines of the frustum. (See in connection with this, § 46). Then lines A A', B B', for instance, may be spoken of as generating lines of the frustum represented.

Similarly, the plans of the longest and shortest generating lines of a cone are contained in the plan, produced, of the axis of its frustum; thus, the plans of V' A and V' B are contained in O O' produced, both ways.

c. The line, produced, which joins the centres of the plans of the ends of a frustum, contains the plan of the apex of the cone; thus, O O', produced, contains V the plan of V'.

d. The line, produced, which joins the plans of corresponding points of a frustum (see definition, § 68) contains the plan of the apex of the complete cone, and, produced only as far as the plan of the apex is the plan of a generating line of the cone; thus, C and C' being corresponding points on the cone, the plan, C C', produced, of the joining C and C', contains V; and C V is the plan of the generating line C V'

e. The plans produced of all generating lines of a frustum intersect the plan of its axis produced, in one point, and that point is the plan of the apex of the complete cone; for example, the plans produced, of the generating lines C C' and D D' of the frustum intersect O O' produced in V.

(70.) It follows from *e* that the plan of the apex of the complete cone of which a given frustum is a portion can easily be found if we have given the plans of the ends of the frustum and the plans of two corresponding points not in the line passing through the centres of the plans of its ends. This is a valuable fact for us, as it spares us elevation drawing which in many cases is very troublesome, and indeed, sometimes practically impossible, as, for instance, where an unequal-tapering body is frustum of an exceedingly high cone the axis of which is but little out of the perpendicular. This is a case in which although the apex cannot be found in elevation because of the great length of the necessary lines, it can readily be found in plan, because, in plan, the requisite lines are short. An oblique cone may of course not only be exceedingly long, but also very greatly out of the perpendicular. In this case it is impracticable anyhow to find the plan of the apex. Problem IV., just solved, meets both cases. It was by *e* that we there found the plan of the apex when accessible, that is, where the lines of the plan are not unduly long (see Fig. 11) by joining the plans of corresponding points *c* and *c'* (*c* and *c'* are corresponding points in that they are mid-points on the half-plans of the ends of the frustum, and therefore necessarily on one and the same generating line), and producing *c c'* to intersect O O', the line joining the centres of the plans of the ends, that is to intersect the plan produced of the axis of the frustum.

(71.) Passing the foregoing under review, it will be seen that if we have two circles which are the plans of the ends of a round unequal-tapering body (frustum of an oblique cone) standing on a horizontal plane, and the circles are in their proper relative positions as part plan of the frustum, then the line produced, one or both ways as may be necessary, which joins the centres of the circles, contains:—

The PLAN of the AXIS of the frustum (see *a*, p. 125).

The PLAN of the AXIS of the cone of which the frustum is a part (see *a*, p. 125).

The PLAN of the APEX of the cone (see *c*, p. 126).

The PLANS of the LONGEST and SHORTEST GENERATING lines of the frustum (see *b*, p. 126).

The PLANS of the LONGEST and SHORTEST GENERATING LINES of the cone, of which the frustum is a part (see *b*, p. 126).

The PLANS of the LINES of GREATEST and LEAST INCLINATION of the frustum (see *b*, p. 126).

The PLANS of the LINES of GREATEST and LEAST INCLINATION of the cone, of which the frustum is a part (see § 52, p. 107).

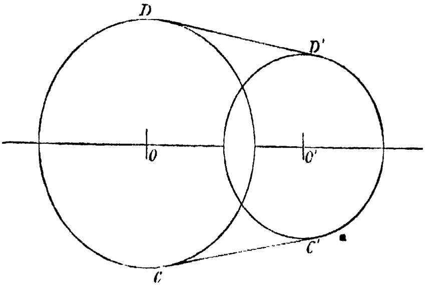
And this is a matter of very great practical importance, as will be seen later on.

(72.) As with circles under the conditions stated, so exactly with arcs which form the plans either of the ends or of portions of the ends, of an unequal-tapering body.

(73.) Further, referring to *c* and *d* of p. 55 as to round equal-tapering bodies we are now in a position to deduce (see Fig. 15) the following as to round unequal-tapering bodies.

f. The plan of a round unequal-tapering body with top and base parallel (frustum of oblique cone) consists essentially of two circles, not concentric, definitely situate relatively to one another. See Fig. 16.

FIG. 16.



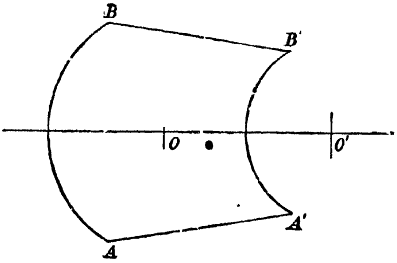
Similarly the plan of a *portion* of such round unequal-tapering body (frustum of oblique cone) consists essentially of two arcs definitely situate relatively to one another, and not concentric. See Fig. 17. O O′ is the axis of the complete frustum.

g. Conversely.—If two circles, not having the same centre, definitely situate relatively to one another, form essentially the plan of a tapering body having parallel ends, that body is a round unequal-tapering body (frustum of oblique cone).

In Fig. 16, if the two circles represent essentially the plan of a tapering body having parallel ends, then the body, of which the circles are the essential plan, is a round unequal-body (frustum of oblique cone).

Similarly if two arcs definitely situate relatively to one another, and not having a common centre, form the essential part of the plan either of a tapering body or of a portion of a tapering body having parallel ends, then that body or portion is a portion of a round unequal-tapering body (frustum of oblique cone).

FIG. 17.



In Fig. 17 if the arcs form the essential part of the plan, either of a tapering body or of a portion of a tapering body having parallel ends; then the body or portion of body, of which that fig. is the plan, is a portion of a round unequal-tapering body (frustum of oblique cone). In the particular plan represented the arcs are similar; the points B and B′, and A and A′ are therefore corresponding points.

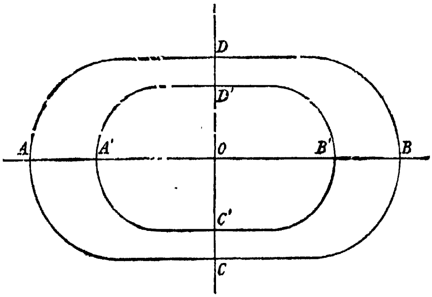
We will now proceed to draw the plans of some unequal-tapering bodies, of which patterns will be presently set out.

PROBLEM VI.

To draw the plan of an unequal-tapering body with top and base parallel and having straight sides and semicircular ends (an "equal-end" bath with semicircular ends), from given dimensions of top and bottom.

Draw A′B′C′D′ (Fig. 18) the plan of the bottom by Problem XVI., p. 20. Bisect A′B′ in O, and through O draw C D perpendicular to A′B′. Make O A and O B each equal to half the length, and O C and O D each equal to half the width of the top. The plan of the top can now be drawn in the same manner as that of the bottom, completing the plan required.

FIG. 18.

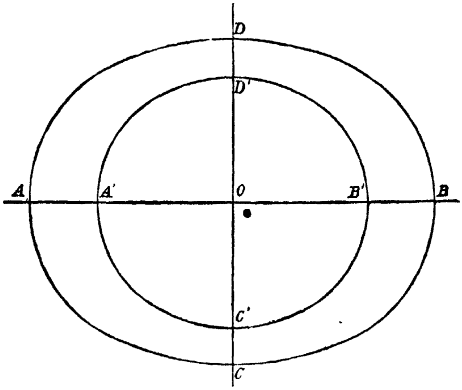


PROBLEM VII.

To draw the plan of an oval unequal-tapering body with top and base parallel (an oval bath), from given dimensions of top and bottom.

Draw (Fig. 19) A′ B′ C′ D′ the plan of the bottom, the given length and width, by Problem XII., p. 13; and make

FIG. 19.



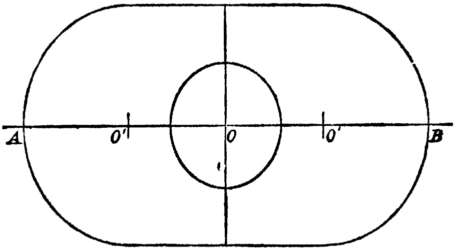
O A and O B each equal to half the length, and O C and O D each equal to half the width of the top. The plan of the top can now be drawn in the same manner as that of the bottom; this completes the plan required.

PROBLEM VIII.

To draw the plan of a tapering body with top and base parallel and having oblong bottom with semicircular ends and circular top (tea-bottle top), from given dimensions of top and bottom.

Draw (Fig. 20) the plan of the oblong bottom by Problem XVI., p. 20, and with O the intersection of the axes of the oblong as centre and half the diameter of the top as radius, describe a circle. This completes the plan.

FIG. 20.



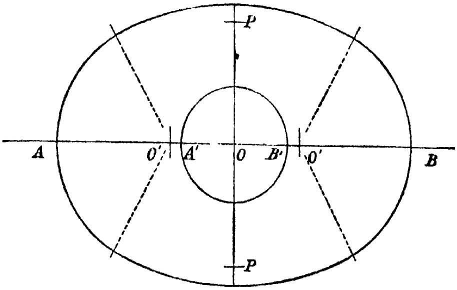
We here, for the first time, extend the use of the word 'axes' (see Problems XII. and XIV., pp. 13 and 15). It is convenient to do so, and the meaning is obvious.

PROBLEM IX.

To draw the plan of a tapering body with top and base parallel, the top being circular and the bottom oval (oval canister-top), from given dimensions of top and bottom.

Draw (Fig. 21) the plan of the oval bottom by Problem XII., p. 13, and with O the intersection of the axes as centre, and half the given diameter of the top as radius, describe a circle. This completes the plan.

FIG. 21

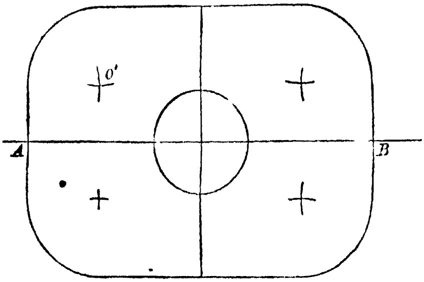


PROBLEM X.

To draw the plan of a tapering body with top and base parallel and having oblong base with round corners and circular top, from given dimensions of the top and bottom.

Draw (Fig. 22) the plan of the oblong bottom by Problem XV., p. 19, and with O the intersection of the axes of the oblong bottom as centre and half the diameter of the top as radius, describe a circle. This completes the plan.

FIG. 22.

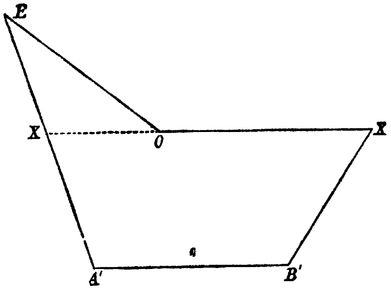


PROBLEM XI.

To draw the plan of an Oxford hip-bath.

Fig. 23 is a side elevation of the bath, drawn here only to make the problem clearer, not because it is necessary for the working. No method that involves the drawing of a full-size side elevation is practical, on account of the amount of space that would be required.

Fig. 23.



The bottom of an Oxford hip-bath is an egg-shaped oval. The portion O X' of the top is parallel to the bottom A' B', and the whole X X' top, the portion O X E of the bath being removed, is also an egg-shaped oval. In speaking of the plan of the 'bath,' we mean the plan of the X X' B' A' portion of it, as the plan of this portion is all that is necessary to enable us to get at the pattern of the bath.

We will first suppose the following dimensions given:—The length and width of the bottom, and the length of the X X' top, the height of the bath in front, and the inclination of the slant at back.

First draw (Fig. 25) the plan of the bottom A' D' B' C' by Problem XIII., p. 14. To draw the horizontal projection of the X X′ top, make (Fig. 24*a*) the angle A A′ E equal to the given inclination of the slant at the back. Through A′ draw A′ H perpendicular to A A′, and equal to the height of the bath in front; through H draw H X parallel to A A′ and cutting A′ E in X; and draw X A perpendicular to X H; then A A′ will be the distance, in plan, at the back, between the curve of the bottom and the curve of the X X′ top (Fig. 23). Make A′ A (Fig. 25) equal to A A′ (Fig. 24*a*), and make A B equal to the length of the X X′ top. With O as centre and O A as radius describe a semicircle; the remainder of the oval of the X X′ top can now be drawn as was that of the bottom. This completes, as stated above, all that is necessary of the plan of the bath to enable its pattern to be drawn.

Fig. 24*a*.

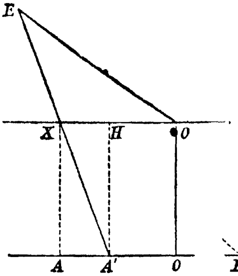
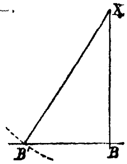


Fig. 24*b*.

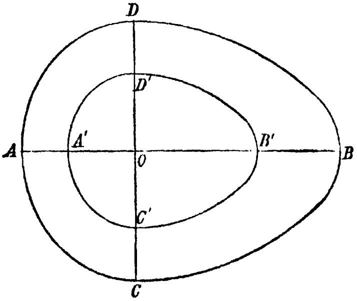


If the width of the X X′ top is given, and not the inclination of the bath at back, make O A (Fig. 25) equal to half that width, and proceed as before. The seam in an Oxford hip-bath, at the sides, is on the lines of which C C′ and D D′ are the plans.

If the length of the X X′ top (Fig. 23) is not given, it can be determined in the following manner:—

Let the angle X B′ B (Fig. 24*b*) represent the inclination of the slant of the front, and B′ X its length. Through X draw X B perpendicular to B′ B; then B B′ will be the distance in plan, at the front, between the curve of the bottom and the curve of the X X′ top (Fig. 23), and this distance, marked from B′ to B (Fig. 25), together with the distance A′ A at the back end of the plan of the bottom, fixes the length required.

FIG. 25.



If the lengths only of the slants of the bath at back and front are given and not their inclinations, the plan of the X X′ top can be drawn as follows:—

Draw two lines X B, B′ B (Fig. 24*b*) perpendicular to one another and meeting in B; make B X equal to the given height of the bath in front, and with X as centre and radius equal to the given length of the slant at the front, describe an arc cutting B′ B in B′. Make B′ B (Fig. 25) equal to B B′ (Fig. 24*b*), and B A equal to the given length of the X X′ top; this will give the distance A′ A. Now make A A′ (Fig. 24*a*) equal to A A′ (Fig. 25), draw A X and A′ H perpendicular to A A′ and equal to the given height of the bath in front; join H X and draw A′ E, through X, equal to the length of the slant at the back; the remainder of the plan of the X X′ top can then be drawn as already described.

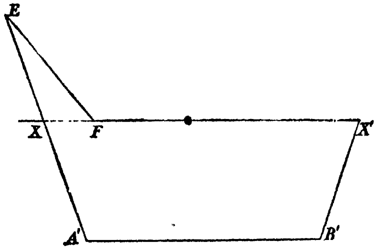
It will be useful to show here in this problem how to complete the back portion already commenced in Fig 24*a* of the side elevation of the bath. Make A′ E equal to the slant (§ 4, p. 24) at back, which must of course be given, and make X O equal to half the given width of the X X′ top; join O E, and draw O O perpendicular to A A′ produced; then A′ E O O is the elevation required.

PROBLEM XII.

To draw the plan of an Athenian hip-bath or of a sitz-bath.

Fig. 26 is a side elevation of the bath, drawn for the reason mentioned in the preceding problem.

FIG. 26.

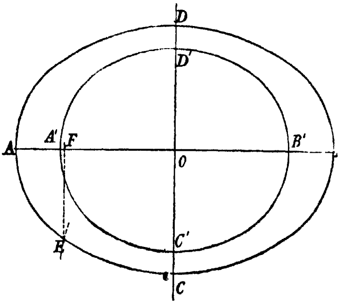


The bottom of an Athenian hip-bath or a sitz-bath is an ordinary oval. The portion X′ F of the top is parallel to the bottom A′ B′, and the whole X X′ top, the portion F X E of the bath being removed, is also an ordinary oval. Similarly as with the bath of the last problem; we mean by plan of the 'bath,' the plan of the X X B′ A′ portion of it; no more being required for the drawing of the pattern of the bath.

We will first suppose the given dimensions to be those of the bottom and the X X′ top of the bath, also height of the bath in front.

First draw A′ D′ B′ C′ (Fig. 27) the plan of the bottom by Problem XII., p. 13. To draw the plan of the X X′ top (Fig. 26) set off O A and O B each equal to half the given length of that top, and O C and O D each equal to half its given width. The plan of the X X′ top can now be drawn

FIG. 27.



as was that of the bottom. This completes, as stated above, all that is necessary of the plan of the bath to enable its pattern to be drawn.

If the length of the X X′ top (Fig. 26) is not given but the inclination of the slant at front and back, these inclinations being the same, the required length can be determined in the following manner:—

Make the angle A A′ E (Fig. 28*a*) equal to the given inclination. Through A′ draw A′ H perpendicular to A A′ and equal to the given height of the bath in front; through H draw H X parallel to A A′ and cutting A′ E in X, and draw X A perpendicular to A A′; then A A′ will be the distance in plan, at back and front, between the curve of the bottom and the curve of the X X′ top. Make A A′ (Fig. 27) and B B′ each equal to A A′ (Fig. 28*a*); then A B will be the length required.

FIG. 28*a*.

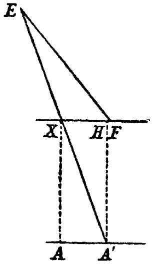
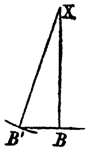


FIG. 28*b*.



If the length of the X X′ top of the bath (Fig. 26) is not given, nor the inclination of the slant at front and back, but only the length of the slant at front, the required length can be thus ascertained.

Draw two lines X B, B′ B (Fig. 28*b*) perpendicular to one another and meeting in B; make B X equal to the given height of the bath in front, and with X as centre, and radius equal to the length of the slant at the front, describe an arc cutting B B′ in B′. Make A′ A and B′ B (Fig. 27) each equal to B B′ (Fig. 28*b*), then A B is the length wanted. The remainder of the plan can be drawn as described above.

By a little addition to Fig. 28*a* we get at the back portion of the side elevation of the bath. It will be useful to do this. Produce A′X and make A′E equal to the slant at back, which must of course be given. Then, on the plan (Fig. 27), E being the meeting point of the end and side curves of the oval A D B C, draw E F perpendicular to A B. Make X F (Fig. 28*a*) equal to A F (Fig. 27); join F E; this completes the elevation required.

PROBLEM XIII.

To draw the plan of an oblong taper bath, the size of the top and bottom, the height, and the slant at the head being given.

To draw D E F C (Fig. 30) the plan of the top. Draw A B equal to the given length of the top, and through A and B draw lines perpendicular to A B. Make A E and A D each equal to half the width of the top at the head of the bath, and B F and B C each equal to half the width of the top at the toe; and join E F and D C. Next from E mark off along E F and E D equal distances, E G and E H, according to the size of the round corner required at the head. (It will be useful practice for the student to work this problem, commencing with the plan of the bottom, and its smaller corners, for the reason given in § 27*a*, p. 63). Through G and H draw lines perpendicular to E F and E D respectively, intersecting in O; and with O as centre and O G as radius describe an arc H G to form the corner. The round corners at D F C, &c., are drawn in like manner.

Fig. 29*a*.

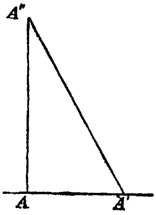
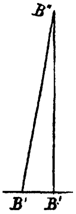


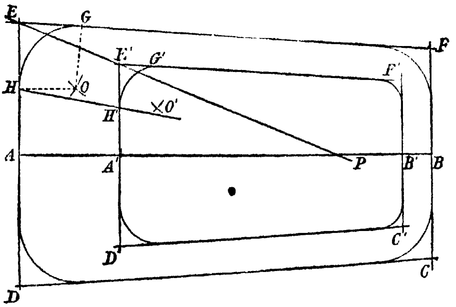
Fig. 29*b*.



To draw the plan of the bottom. Let the angle A" A' A (Fig. 29*a*) be the angle of the inclination of the slant at the head, and A' A" the length of the slant. Through A" draw

A″ A perpendicular to A A′, then A A′ will be the distance between the lines, in plan, of the top and bottom at the head. Make A A′ (Fig. 30) equal to A A′ (Fig. 29*a*), and A′ B′ equal to the length of the bottom. Through A′ and B′ draw lines each perpendicular to A B; make A′ E′ and A′ D′ each equal to half the width of the bottom at the head, and B′ F′ and B′ C′ each equal to half the width of the bottom at the toe. Join E′ F′ and D′ C′. The round corner of the bottom at the head must be drawn in proportion to the round corner of the top at the head, and this is done in the following manner. Join E E′ and produce it, to meet A B in P, and join H P by a line cutting D′ E′ in H′; make E′ G′ equal to E′ H′, and complete the corner from centre O′ obtained as was the centre O. Draw the other corners in similar way, and this will complete the plan required. The D corner is like the E corner; the corners also at F and C correspond. Similarly with the E′ and D′, and F′ and C′ corners.

Fig. 30.



If the length of the bath is given and the length of slant at (but not its inclination) head or toe, the distance A A′ can be found by drawing two lines A″ A, A′ A (Fig. 29*a*) perpendicular to one another and meeting in A, and making A A″ equal to the given height; then, with A″ as centre and A″ A′, the given length of the slant at the head, as radius, describe an arc cutting A A′ in A′. Then A A′ is the distance required. Similarly (Fig. 29*b*) the distance B B′ can be found.

CHAPTER III.

PATTERNS FOR ARTICLES OF UNEQUAL TAPER OR INCLINATION OF SLANT, AND HAVING FLAT (PLANE) SURFACES.

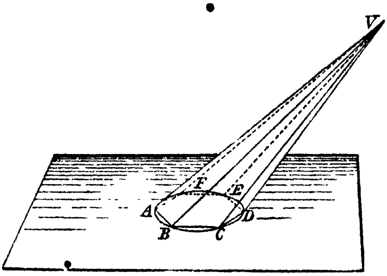
(CLASS II. *Subdivision b.*)

ARTICLES of unequal taper or inclination of slant, and having plane or flat surfaces (hoppers, hoods, &c.), are frequently portions (*frusta*) of oblique pyramids, or parts of such *frusta*.

DEFINITIONS.

(74.) Oblique Pyramid: Frustum of Oblique Pyramid:—Oblique pyramids have not yet been defined. For our purpose it will be sufficient to define an oblique pyramid negatively, that is, as a pyramid which is not a right pyramid; and when cut by a plane parallel to its base (that is when truncated), to define its frustum (§ 33, p. 69) as the frustum of a pyramid which is not a right pyramid. In the oblique pyramid the faces are not all equally inclined. Articles of which the faces are not all equally inclined are

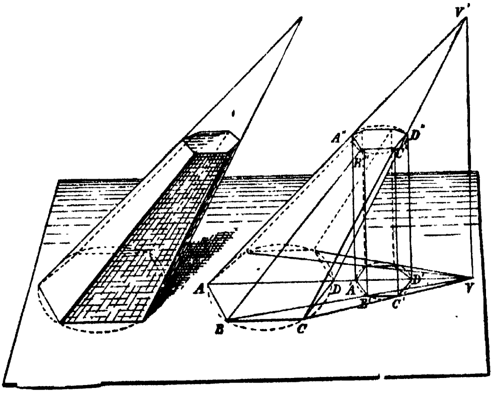
FIG. 31.



not necessarily portions of oblique pyramids. One such case will be given later on. The problems immediately following deal with articles of which the faces are not all equally inclined, but which are portions of oblique pyramids.

(75.) Further, an oblique pyramid, when it has a base through the angular points of which a circle can be drawn, can be inscribed in an oblique cone like as a right pyramid in a right cone, and this property gives constructions for solving most of our oblique-pyramid problems, somewhat similar to those in Book I., Chapter VI., where the right pyramid is concerned. Fig. 31 represents an oblique hexagonal pyramid inscribed in an oblique cone. This fig. should be compared with Fig. 31, p. 67. The edges of the oblique pyramid are generating lines of the cone.

FIG. 32.



(76.) Also from Fig. 32 it will be seen that the plan of a frustum of an oblique hexagonal pyramid standing on a horizontal plane consists of two hexagons A B C D and A' B' C' D' (the plans of the ends), whose similarly situated sides, A B and

A′ B′, B C and B′ C′, C D and C′ D′ for instance, are parallel, and whose corresponding points (§ 68, p. 124) A, A′ and B, B′, for instance, are joined by lines A A′, B B′, which are the plans of the edges of the frustum. Just as in the case of the frustum of the oblique cone (see *d* and *e*, p. 126), if a line joining corresponding points in plan be produced, it will contain the plan of the apex of the complete pyramid of which the frustum is a portion; and if another such line be produced to intersect the first line, the point of intersection will be that plan of apex. For example, the lines A A′, B B′ and C C′ produced meet in a point which is the plan of the apex of the pyramid of which the frustum A B D D″ B″ A″ is a portion.

(77.) From this it follows that if the plan of a tapering body with top and base parallel and having plane or flat surfaces be given, we can at once determine whether the tapering body is or not a frustum of an oblique pyramid by producing the plans of the edges. If these meet in one point, then the given plan is that of a frustum of an oblique pyramid.

PROBLEM XIV.

To draw the pattern of an oblique pyramid.

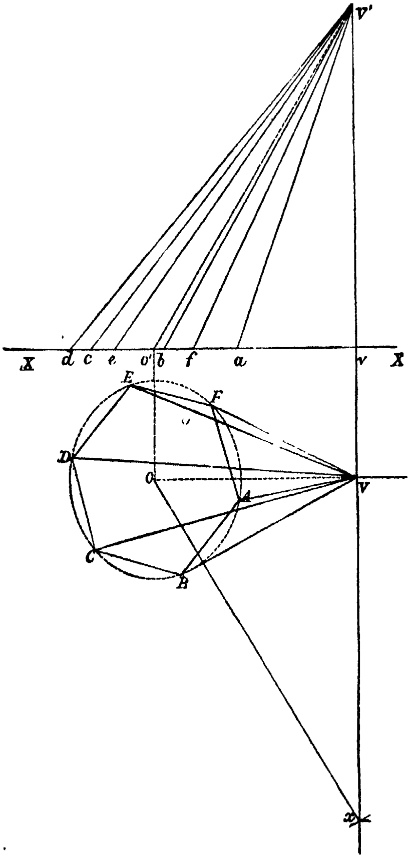
Case 1.—Given the plan of the pyramid and its height.

Let A B C D E F V (Fig. 33) be the plan of the pyramid (here a hexagonal pyramid), V being the apex, and O V the plan of the axis. Draw X X parallel to O V, and through V draw V V′ perpendicular to X X, and cutting it in *v*; make *v* V′ equal to the given height of the pyramid. Next make ***v a***, ***v b***, *v c*, *v d*, *v e*, and *v f* equal respectively to V A, V B, V C, V D, V E, and V F, the plans of the edges of the pyramid. Joining V′ ***a***, V′ *b*, V′ *c*, &c., will give the true lengths of these edges.

To draw the pattern of the pyramid with the seam at the edge V A. Draw V A (Fig. 34) equal to V′ *a* (Fig. 33); with

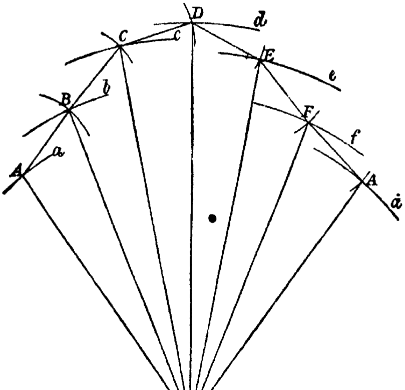
/ as centre and V′ b (Fig. 33) as radius describe arc b, and with \ as centre and A B (Fig. 33) as radius describe an arc interecting the arc b in B. The other points C, D E, F, A, are

Fig. 33



found in similar manner. Thus, with V (Fig. 34) as centre and V′ *c*, V′ *d*, V′ *e*, V′ *f*, and V′ *a* (Fig. 33) successively as radii, describe arcs *c*, *d*, *e*, *f*, and *a* (Fig. 34). Next, with B (Fig. 34) as centre and B C (Fig. 33) as radius, describe an arc intersecting arc *c* in C; with C D (Fig. 33) as radius and C (Fig. 34) as centre describe an arc cutting arc *d* in D; with D (Fig. 34) as centre and D E (Fig. 33) as radius describe an arc intersecting arc *e* in E; and so on for points F and A. Join A B, B C, C D, DE, E F, F A and A V, and this will complete the pattern required.

FIG. 34.



Joining the points B, C, D, &c., to V, it will be seen that the pattern is made up of a number of triangles, each triangle being of the shape of a face of the pyramid, also that the construction of the pattern is very similar to the construction of that of an oblique cone.

Should it be inconvenient to draw X X in the position shown in Fig. 33, the true lengths of the edges of the pyramid can be found in the following manner. Draw X X quite apart from the plan of the pyramid, and from any point v in it draw v V′ perpendicular to X X, and equal to the height of the pyramid, and proceed as just described.

CASE II.—Given the plan of the pyramid and the length of its axis.

Draw X X (Fig. 33) parallel to O V, the plan of the axis; through V draw V V′ perpendicular to X X, and through O draw O O′ perpendicular to X X. With O′ as centre and the given length of the axis as radius describe an arc cutting V V′ in V′; then v V′ will be the height of the pyramid, and we now proceed as in Case 1.

Or, draw V x perpendicular to O V, and with O as centre and radius equal to the length of the axis describe an arc cutting V x in x; V x will be the height of the pyramid.

PROBLEM XV.

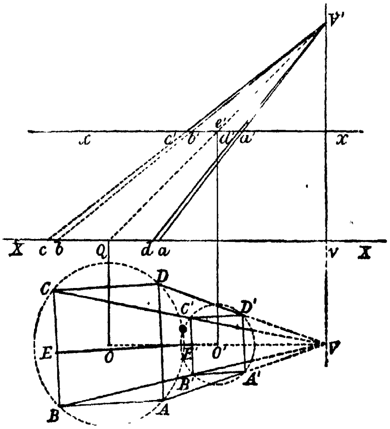
To draw the pattern of a frustum of an oblique pyramid.

CASE I.—Given the plan of the frustum and its height.

Let A B C D D′ A′ B′ C′ (Fig. 35) be the plan of the frustum (here of a square pyramid). Produce A A′, B B′, &c., the plans of the edges to meet in a point V; this point is the plan of the apex of the pyramid of which the frustum is a part. Join O, the centre of the square which is the plan of the large end of the frustum, to V. The line O V will pass through o', the centre of the plan of the small end; O O′ will be the plan of the axis of the frustum, and O V the plan of the axis of the pyramid of which the frustum is a portion.

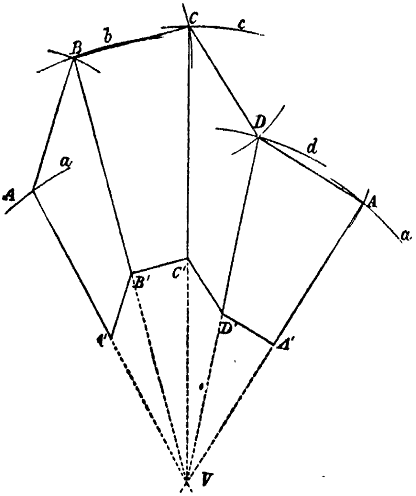
Draw X X parallel to O V; through V draw V V′ perpendicular to X X, and cutting it in v. Make $v\,x$ equal to the given height of the frustum, and through x draw $x\,x$ parallel to X X; through O draw O Q perpendicular to X X and meeting it in Q and through O′ draw O′ Q′ perpendicular to X X and cutting *x x* in *e*′. Join Q *e*′ and produce it to intersect *v* V′ in V′. Next make *v a*, *v b*, *v c*, *v d* equal to V A, V B, V C, V D respectively; join *a*, *b*, *c*, and *d* to V′ by lines cutting *x x* in points *a*′, *b*′, *c*′, and *d*′; *a a*′, *b b*′, &c., are the lengths of the edges of the frustum.

FIG. 35.



To draw the pattern with the seam at A A′. Draw V A (Fig. 36) equal to V′ *a* (Fig. 35); with V as centre and V′ *b* (Fig. 35) as radius describe an arc *b*, and with A as centre and A B (Fig. 35) as radius describe an arc intersecting arc *b* in B; with V′ *c* (Fig. 35) as radius and V as centre describe arc *c*, and with B C (Fig. 35) as radius and B as centre describe an arc intersecting the arc *c* in C. Next with V′ *d* and V′ *a* (Fig. 35) as radii and V as centre describe arcs *d* and *a*; with C as centre and radius C D (Fig. 35) describe an arc intersecting arc *d* in D; and with D A (Fig. 35) as radius and D as centre describe an arc intersecting the arc *a* in A. Join A, B, C, D, and A to V; make A A′, B B′, C C′, D D′ respectively equal to $a a'$, $b b'$, $c c'$, $d d'$ (Fig. 35), and join A B, B C, C D, D A, A′ B′, B′ C′, C′ D′, &c. Then A B C D A A′ D′ C′ B A′ is the pattern required.

FIG. 36.



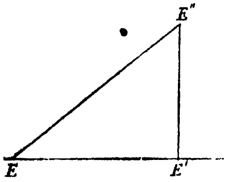
(78.) The dotted circles (Fig. 35) through the angular points of the plans of the ends show the plans of the ends of the frustum of the oblique cone which would envelop the frustum of the pyramid. From the similarity of the construction above to that for the pattern of a frustum of an oblique cone, it will be evident that we have treated the edges of the frustum as generating lines (see *b*, p. 126) of the frustum of the oblique cone in which the frustum of the pyramid could be inscribed.

Should it be inconvenient to draw X X in conjunction with the plan of the pyramid draw X X quite apart, and from any point *v* in it draw *v* V′ perpendicular to X X; make *v x* equal to the height of the frustum and draw *x x* parallel to X X. Make *v a*, *v b*, *v c*, *v d* equal to V A, V B, V C, V D (Fig. 35) respectively; and make *x a′*, *x b′*, *x c′*, *x d′* equal to V A′, V B′, V C′, V D′ (Fig. 35) respectively. Join *a a′*, *b b′*, *c c′*, and *d d′* by lines produced to meet *v* V′ in V′, and proceed as stated above.

CASE II.—Given the dimensions of the two ends of the frustum, the slant of one face and its inclination (the slant of the face of a frustum of a pyramid is a line meeting its end lines and perpendicular to them).

Draw (Fig. 37) a line E E″ equal to the given slant, make the angle E″ E E′ equal to the given inclination, and let fall E″ E′ perpendicular to E E′. Draw A B C D (Fig. 35), the plan of the large end of the frustum, and let B C be the plan of the bottom edge of the face whose slant is given. Bisect B C in E and draw E E′ perpendicular to B C and equal to E E′ (Fig. 37). Through E′ draw B′ C′ parallel to B C; make E′ C′ and E′ B′ each equal to half the length of the top edge of the B C face, through C′ and B′ draw C′ D′ and B′ A′ parallel to C D and B A; make C′ D′ and B′ A′ each equal to B′ C′; join D′ A′, also A A′, B B′, C C′, and D D′; this will complete the plan of the frustum. E′ E″ (Fig. 37) is the height of the frustum. The remainder of the construction is now the same as that of Case I.

FIG. 37.



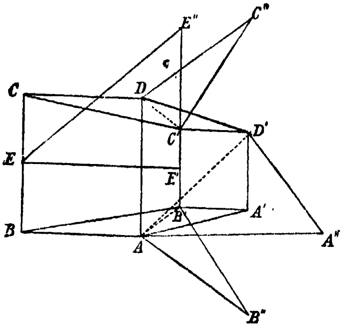
For large work and where the ends of a frustum are of nearly the same size, it would be inconvenient to use long radii. For unequal-tapering bodies which are not portions of oblique pyramids, as in Problem XVII., the method now given, or a modification of it, must be used.

PROBLEM XVI.

To draw, **without long radii,** *the pattern for a frustum of an oblique pyramid. The plan of the frustum and its height being given.*

Let A B C D D' A' B' C'' (Fig. 38) be the plan of the frustum. From any point E in B C draw E E' perpendicular to B C and B' C', the plans of the bottom and top edges of the face B C B' C' of the frustum. Draw E' E'' perpendicular to E E' and equal to the height (which either is given or can be found as in Case II. of last problem), and join E E'', then E E'' is the true length of a slant of the face B C B' C', of the frustum. Join D C', and find its true length (D C'') by drawing C' C''

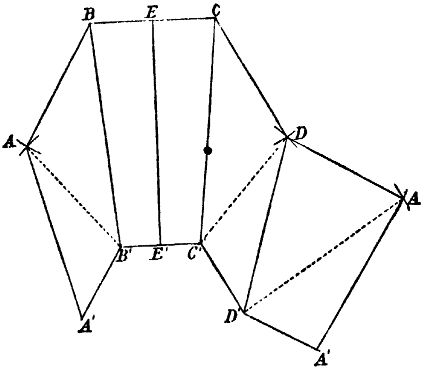
FIG. 38.



perpendicular to D C'' and equal to the height of frustum and joining D C''. Next join D' A and B' A; through D' and B' draw lines D' A'', B' B'' perpendicular to D' A and B' A respectively, and make D' A'' and B' B'' each equal to the given height of the frustum; join A A'' and A B'', then A A'' and A B'' are the true lengths of D' A and B' A respectively.

To draw the pattern of the face B C B' C', draw E E' (Fig. 39) equal to E E'' (Fig. 38), and through E and E' draw B C and B' C' perpendicular to E E'. Make E C, E B, E' C', and E' B' equal to E C, E B, E' C', and E' B' (Fig. 38) respectively; join C C' and B B'; this completes the pattern of the face. The patterns of the other faces are found in the following manner:—With C' (Fig. 39) and C as centres and

FIG. 39.



D C'' and C D (Fig. 38) as radii respectively, describe arcs intersecting in D; join C D, draw C' D' parallel to C D and equal to C' D' (Fig. 38); and join D D'. With D' and D (Fig. 39) as centres and A A'' and D A (Fig. 38) as radii respectively describe arcs intersecting in A; join D A, draw D' A' parallel to D A and equal to D' A' (Fig. 38), and join A to A′. Next, with B′ and B as centres and A B″ and B A (Fig. 38) respectively as radii, describe arcs intersecting in A; join B A and draw B′ A′ parallel to B A and equal to B′ A′ (Fig. 38). Join A A′, and this will complete the pattern required.

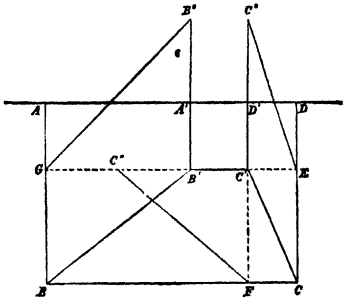
PROBLEM XVII.

To draw the pattern for a hood.

The plan of the hood is necessarily given, or else dimensions from which to draw it. Also the height of the hood, or the slant of one of its faces. The hood is here supposed to be a body of unequal taper with top and base parallel, but not a frustum of an oblique pyramid.

Let A B C D A′ B′ C′ D′ (Fig. 40) be the given plan of the hood (a hood of three faces), A D being the 'wall line,' A B and D C perpendicular to A D and B C parallel to it, also let the length of F C″, a slant of face B B′ C′ C, be given. Draw C′ F perpendicular to B C and through C′ draw C′ C″ perpendicular to C′ F, and with F as centre and radius

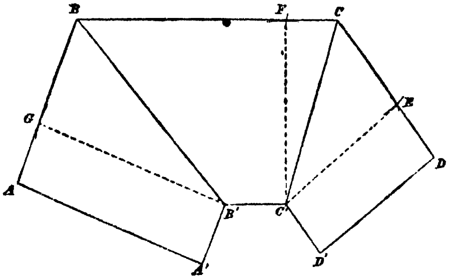
FIG. 40.



equal to the given length, describe an arc cutting C′ C″ in C″. Join F C″; then C′ C″ is the height of the hood, which we need. If the height of the hood is given instead of the length F C″, make C′ C″ equal to the height and join F C″, which will be the true length of F C. Next, through C′ draw C′ E perpendicular to C D; draw C′ C″ perpendicular to C′ E, make C′ C″ equal to the height and join E C″. Now produce C′ B′ to meet A B in G; draw B′ B″ perpendicular to B′ G and equal to the height, and join G B″.

To draw the pattern of the hood. Draw F C′ (Fig. 41) equal to F C″ (Fig. 40); through F and C′ draw B C and B′ C′, each perpendicular to F C′; make F B equal to F B (Fig. 40); make F C equal to F C (Fig. 40), and C′ B′ equal to C′ B′ (Fig. 40). Join B B′ and C C′, then B B′ C′ C will be the pattern of the face of which B B′ C′ C (Fig. 40) is the plan. To draw the pattern of the face C′ D′ D C (Fig. 40). With C′ and C (Fig. 41) as centres and E C″ and C E (Fig. 40) as radii respectively, describe arcs intersecting in E. Join C E and produce it, making C D equal to C D (Fig. 40), and through C′ draw C′ D′ parallel to C D and equal to C′ D′ (Fig. 40). Join D D′, then C′ C D D′ is the pattern of the face of which C′ C D D′ (Fig. 40) is the plan. With B′ and B as

FIG. 41.



centres and radii respectively equal to B″ G and B G (Fig. 40), describe arcs intersecting in G. Join B G and produce it, making B A equal to B A (Fig. 40), and through B′ draw B′ A′ parallel to B A and equal to B′ A′ (Fig. 40). Join A A′; and the pattern for the hood is complete.

CHAPTER IV.

PATTERNS FOR UNEQUAL-TAPERING ARTICLES OF FLAT AND CURVED SURFACE COMBINED.

CLASS II. (*Subdivision c.*)

FROM what has been stated about the plans of unequal-tapering bodies and from *g*, p. 129, it will be evident that the curved surfaces of the articles now to be dealt with are portions of frusta of oblique cones.

(79.) The advantages referred to in § 61 of looking upon the oblique cylinder as frustum of an oblique cone will be evident in this chapter. For there is to each of the problems a Case where the plan arcs of the curved portions of the body treated of have equal radii. To deal with these as problems exceptional to a general principle would be most inconvenient. As extreme cases, however, of the one principle that the curved portions of the bodies before us are portions of frusta of oblique cones, their solution presents no difficulty. It will be sufficient to take one such Case in connection with only one of the bodies. This we shall do in Case IV. of the next problem.

PROBLEM XVIII.

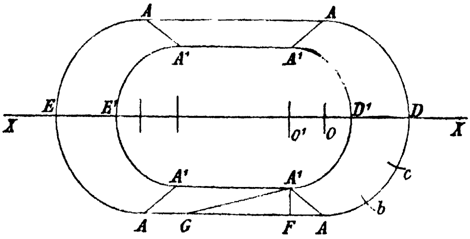
To draw the pattern for an unequal-tapering body with top and base parallel and having flat sides and semicircular ends (an 'equal-end' bath, for instance), the dimensions of top and bottom of the body and its height being given.

Five cases will be treated of; four in this problem and one in the problem following.

CASE I.—Patterns when the body is to be made up of four pieces.

Draw (Fig. 42) the plan of the body (see Problem VI., p. 130), preserving of its construction the centres O, O′ and the points A, A′ in which the plan lines of the sides and curves of the ends meet each other. Join A A′, as shown (four places) in the fig. The ends A D A A′ D′ A′ and A E A A′ E′ A′ of the body (see *g*, p. 129) are portions of frusta of oblique cones. Let us suppose that the seams are to be at the four A corners where they are usually placed, and to correspond with the four lines A A′. Then we shall require one pattern for the flat sides, and another for the semicircular ends.

FIG. 42.



To draw the end pattern.

(80.) Draw A *b* D D′ A′ (Fig. 43) the A *b* D D′ A′ portion of Fig. 42 separately, thus. Draw any line X X and with any point O (to correspond with O, Fig. 42) in it as centre and O D (Fig. 42) as radius describe an arc (here a quadrant) D *b* A equal to the arc D *b* A (Fig. 42). Make D O′ equal to D O′ (Fig. 42), and with O′ as centre and O′ D′ (Fig. 42) as radius describe an arc (here a quadrant) D′ A′ equal to the arc D′ A′ (Fig. 44). Joining A A′ completes the portion of Fig. 42 required. Now divide D A into any number of equal parts, here three, in the points *b* and *c*. From D′ draw D′ D″ perpendicular to X X and equal to the given height. Then D, D″ are, in elevation, the corresponding points of which

D, D′ are the plans. Being corresponding points, they are in one and the same generating line (§ 68). Join D D″ and produce it indefinitely, then somewhere in that line will lie the elevation of the apex of the cone of a portion of which

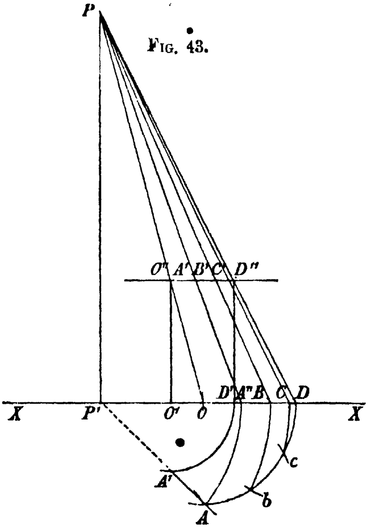
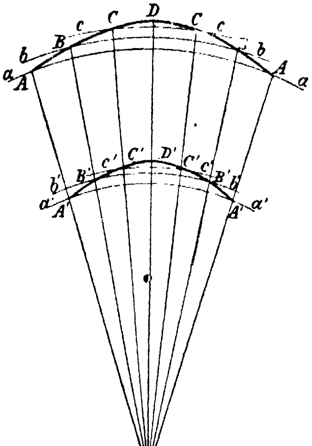


FIG. 43.

A *b* D D′ A′ is the plan. Now from O′ draw O′ O″ perpendicular to X X and equal to the given height. Then O, O″ are, in elevation, the centres of the ends of the frustum in the same plane that D, D″ are represented in, that is in the plane of the paper; O, O′ being the plans of these centres. Join O O″ and produce it indefinitely; then in this line lies the elevation of the axis of the cone of a portion of which A *b* D D′ A′ is the plan, and necessarily therefore the elevation of the apex. That is, the intersection point P of these two lines is the elevation of the apex. Next, from P let fall P P′ perpendicular to X X; then P′ will be the plan of the apex. Join D″ O″. With P′ as centre and P′ *c*, P′ *b*, and P′ A successively as radii, describe arcs cutting X X in C, B, and A″. Join these points to P by lines cutting O″ D″ in C′, B′, and A′.

Next draw a line P D (Fig. 44) equal to P D (Fig. 43), and with P as centre and P C, P B, and P A″ (Fig. 43) successively as radii describe arcs *c c*, *b b*, and *a a*. With D as centre and radius equal D *c* (Fig. 43) describe arcs cutting arc *c c* in C and C right and left of P D. With same radius and these points C and C successively as centres describe arcs cutting arc *b b* in B and B right and left of P D. With B and B successively as centres and same radius describe arcs cutting arc *a a* in A and A right and left of P D. Join the points C, B, and A right and left of P D to P. With P as centre and P D″ (Fig. 43) as radius describe an arc cutting P D in D″. With same centre, and P C′ (Fig. 43) as radius, describe arc *c′ c′* cutting lines P C right and left of P D in C′ and C′. With same centre, and P B′ (Fig. 43) as radius, describe arc

FIG. 44.

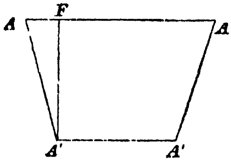


$b'b'$ cutting lines P B right and left of P D in B′ and B′. Similarly find points A′ and A′. Through the successive points A, B, C, D, C, B, A, draw an unbroken curved line. Also through the successive points A′, B′, C′, D′, C′, B′, A, draw an unbroken curved line. Then A D A A′D′ A′ will be the required pattern for ends of the body.

To draw the pattern for the sides.

Through A′ (Fig. 42) draw A′ F perpendicular to A′ A′ make F G equal to the given height and join A′G. Then A′G is the slant of the body at the side. Next draw (Fig. 45) A A equal to A A (Fig. 42), and make A F equal to A F (Fig. 42); through F draw F A′ perpendicular to A A and equal to A′G (Fig. 42), and through A′ draw A′ A′ parallel to A A. Make A′ A′ equal to A′ A′ (Fig. 42). Join A A′, A A′, then A A′ A′ A is the pattern for the sides.

Fig. 45.



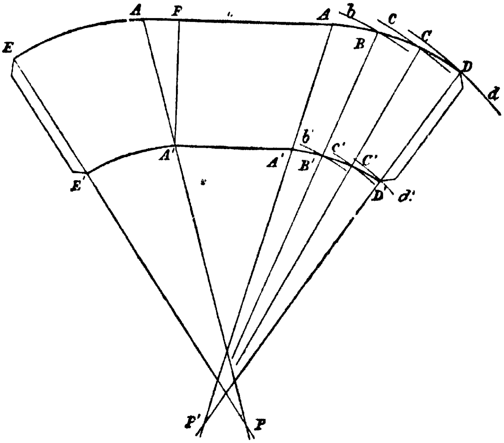
CASE II.—Pattern when the body is to be made up of two pieces.

We will take it that the seams are to be at D D′ and E E′ (Fig. 42). It is evident that we want but one pattern, which shall include a side of the body and two half-ends.

First draw as just explained A′ A F A A′ (Fig. 46) a side-pattern of the body. Produce one of the lines A A′ of this pattern, and make A P′ equal to A″P (Fig. 43). With P′ as centre and P B, P C, and P D (Fig. 43) successively as radii describe arcs *b*, *c*, and *d*, and with A as centre and A *b* (Fig. 43) as radius describe an arc cutting arc *b* in B. With same radius and B as centre describe an arc cutting arc *c* in C; similarly with C as centre and same radius find D

Join B P′, C P′, D P′. Now with P B′ (Fig. 43) as radius and P′ as centre describe an arc b′ cutting P B in B′, and with P C′, P D″ (Fig. 43) successively as radii describe arcs c′ and d cutting P′ C and P′ D in C′ and D′. Through the points A, B, C, and D draw an unbroken curved line. Also through the points A′, B′, C′, and D′ draw an unbroken curved line. Then A D D′ A′ will be a half-end pattern attached to the right of the side pattern. Draw the other half-end pattern A E E′ A′ in the same manner; then E F D D′ A′ A′ E′ will be the complete pattern required.

Fig. 46.

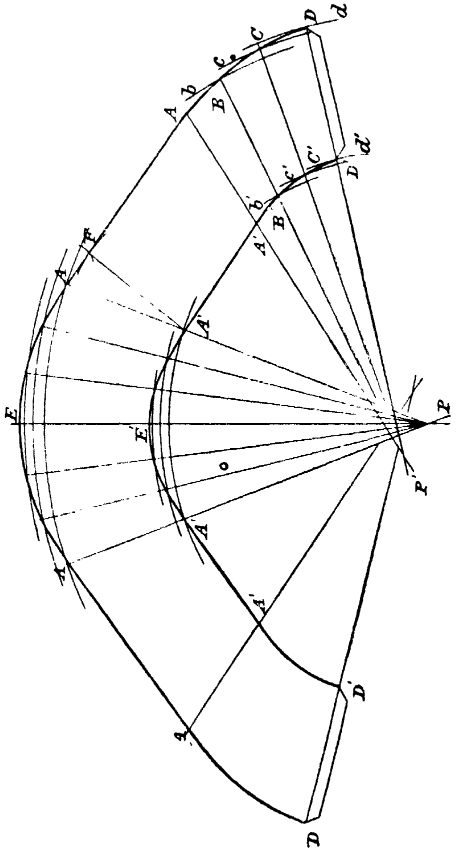


Case III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with D D′ (Fig. 42).

First draw A E A A′ E′ A′ (Fig. 47) an end pattern of the body in the same manner that A D A A′ D′ A′ (Fig. 44) was drawn. With A′ and A (right of E E′) as centres and A′ G and A F (the small length A F) (Fig. 42) respectively as radii describe arcs intersecting in F; join A F and produce it, making A A equal to A A (Fig. 42). Through A′ (extremity of F A′) draw A′ A′ parallel to A A and equal to

FIG. 47

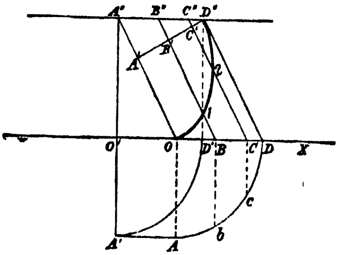


A′ A′ (Fig. 42). Join A A′, the extremities of the lines just drawn, and produce it indefinitely; and make A P′ equal A″ P (Fig. 43). With P″ as centre and P B, P C, and P D (Fig. 43) successively as radii describe arcs *b*, *c*, and *d*, and with A (of A P′) as centre and A *b* (Fig. 43) as radius describe an arc cutting arc *b* in B. With same radius and B as centre describe an arc cutting arc *c* in C, and similarly with C as centre and same radius find D. Join B P′, C P′, and D P′. Now with P′ as centre and radii successively equal to P B′, P C″, and P D″ (Fig. 43) describe arcs *b*′, *c*′, and *d*′ cutting P′ B, P′ C, and P′ D in B′, C″, and D′. Through the points A, B, C, and D draw an unbroken curved line. Also through the points A′, B′, C′, and D′ draw an unbroken curved line. We have now in D F A A′ A′ D′ attached to the right of the end pattern we started with, a side pattern and a half-end pattern. By a repetition of the foregoing construction we can attach A A D D′ A′ A′ to the left of the end pattern we started with. The figure D E D D′ E′ D′ will be the complete pattern required.

CASE IV.—Where the plan arcs D A, D′ A′ (Fig. 42) have equal radii.

This is the extreme case above (§ 79, p. 157) referred to, where the cone becomes cylindrical. Problem V., p. 121, may

FIG. 48.

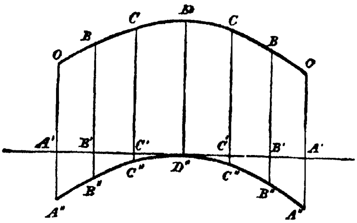


advantageously be compared with the work now given. The arcs (Fig. 48) D A and D′ A′ (here quadrants) being equal their radii O D, O' D' are also equal. Through D' and O' draw D' D", and O' A" perpendicular to X X and each equal to the given height of the body. Join D D", A" D"; divide the arc D A into any number of equal parts, here three, in the points *b* and *c*; and through *c*, *b*, and A draw *c* C, *b* B, and A O each perpendicular to X X and cutting it in C, B, and O respectively. The arc D A being here a quadrant the point where the line from A perpendicular to X X cuts X X is necessarily O, the centre whence the arc is drawn. Through C, B, and O draw C C", B B", and O A" parallel to D D" and cutting A" D" in points C", B", and A". Also through D" draw a line D" A' perpendicular to D D" and cutting the lines just drawn in C', B' and A'. Make *c* 2 equal to C *c*; B' 1 equal to B *b*, and A' O equal to O A. From D" through 2, 1, to O draw an unbroken curved line.

To draw the pattern.

Draw D D" (Fig. 49) equal to D D" (Fig. 48) and through D" draw an indefinite line A' A' perpendicular to D D".

FIG. 49.



Mark off on A' A', right and left of D", D" C', C' B', and B A' respectively equal to the distances between D" and 2, 2 and 1, and 1 and 0 (Fig. 48); and through C', B', and A', right and left of D" draw indefinite lines each parallel to D D". Make C' C right and left of D D" equal to C' C (Fig. 48); and make B' B right and left of D D" equal to B' B (Fig. 48). Also make A' O right and left of D D" equal to A' O (Fig. 48).

Next make C′C″ right and left of D D″ equal to C′C″ (Fig. 48). Similarly find points B″, A″ right and left of D D″ by making B′B″, A′A″ respectively equal to B′B″, and A′A″ (Fig. 48). Through the points O, B, C, D, C, B, O, draw an unbroken curved line. Also through the points A″, B″, C″, D″, C″, B″, A″, draw an unbroken curved line. Then O D O A″ D″ A″ will be the pattern required.

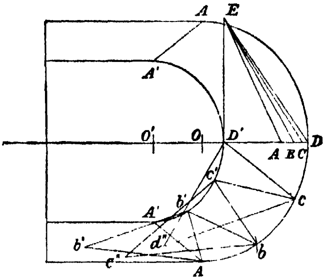
PROBLEM XIX.

To draw, without long radii, *the pattern for an unequal-tapering body with top and base parallel and having flat sides and equal semicircular ends (an 'equal-end' bath, for instance). The dimensions of the top and bottom of the body and its height being given.*

This problem is a fifth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern.

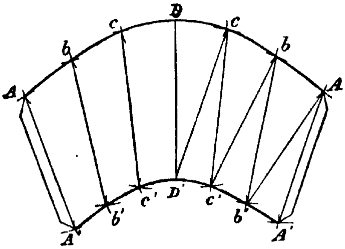
Fig. 50.



First draw half the plan (Fig. 50). It is evident that the drawing of the side pattern presents no difficulty, as long radii are not involved. It can be drawn as in Case I. of preceding problem. Divide the quadrants D A, D′ A′, each into the same number of equal parts, here three, in the points *c*, *b*, *c*′, *b*′; join *c c*′, *b b*′. Through D′ draw D′ E perpendicular to D′ D and equal to the given height of the body. From D′ along D′ D mark off D′ A, D′ B, and D′ C respectively equal to A′ A, *b*′ *b*, and *c*′ *c*; and join points D, C, B, and A to E, then E A, E B, E C, and E D, will be the true lengths of A′ A, *b*′ *b*, *c*′ *c*, and D′ D respectively. Next join *c* D′, and draw D′ *d*″ perpendicular to D′ *c* and equal to the given height. Join *c d*″, then *c d*″ may be taken as the true length of D′ *c*. Similarly join *b c*′ and A *b*′, and through *c*′ and *b*′ draw *c*′ *c*″ and *b*′ *b*″ perpendicular to *c*′ *b* and *b*′ A respectively, and each equal to the given height. Join *b c*″ and A *b*″, then *b c*″ and A *b*″ may be taken as the true lengths of *b c*′ and A *b*′ respectively.

Now draw (Fig. 51) a line D D′ equal to D E (Fig. 50) and with D′ and D as centres and radii respectively equal to *d*″ *c* and D *c* (Fig. 50) describe arcs right and left of D D′, intersecting in *c* and *c*. With *c*, right of D, and D′ as centres, and radii respectively equal to C E and D′ *c* (Fig. 50) describe arcs intersecting in *c*′, right of D′. With *c*, left of D, and D′ as centres, and radii as before, describe arcs intersecting in *c*′, left of D′. With successively *c*′ and *c* right and left of D D′,

FIG. 51.



as centres and radii respectively equal to $c''b$ and cb (Fig. 50) describe arcs intersecting in b and b right and left of D D'. With successively b and c' right and left of D D' as centres and radii respectively equal to B E and $c'b'$ (Fig. 50) describe arcs intersecting in b', and b' right and left of D D', and with successively b' and b right and left of D D' as centres and radii respectively equal to A b'' and b A (Fig. 50) describe arcs intersecting in A and A right and left of D D'. Similarly with A E and b' A' (Fig. 50) as radii and centres respectively A and b' describe intersecting arcs to find points A' and A', right and left of D' D. Through the points A, b, c, D, c, b, A draw an unbroken curved line. Also through the points A', b', c D', c', b', A', draw an unbroken curved line. Join A A', A A', right and left of D D', then A D A A' D' A' will be the pattern required.

The lines cc', c D', bb', &c., are not needed in the working; they are drawn here to aid the student by showing him how the pattern corresponds with the plan, line for line of same lettering (see also § 65, p. 121).

PROBLEM XX.

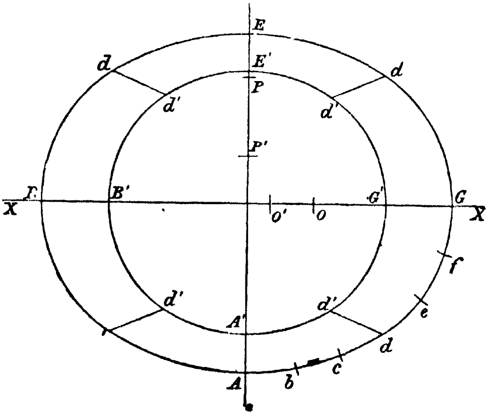
To draw the pattern for an oval unequal-tapering body with top and base parallel (an oval bath, for instance). The height and dimensions of the top and bottom of the body being given.

Four cases will be treated of, three in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 52) the plan of the body (see Problem VII., p. 131), preserving of its construction, the centres O, O', P, P' and the several points d and d' in which the side and end curves meet each other. Join dd', as shown (four places) in the fig. From the plan we know (see g, p. 129) that d G d d' G' d', d B dd' B' d', the ends of the body are like portions of the frustum of an oblique cone; we also know that *d* A *d d'* A′ *d'*, *d* E *d d'* E′ *d'*, the sides of the body are like portions of the frustum of an oblique cone.

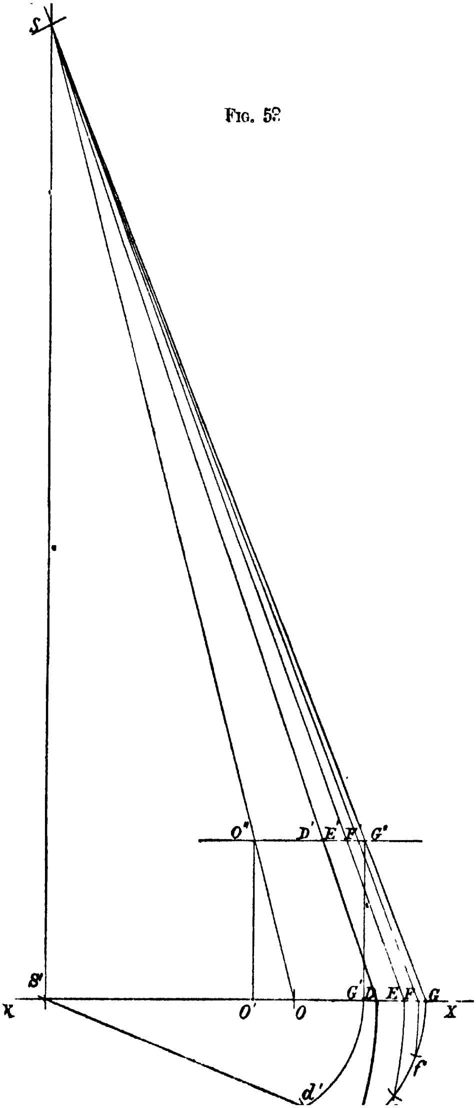
In Plate I. (p. 181), is a representation of the oval unequal-

FIG. 52.



tapering body for which patterns are required, also of two oblique cones (*x* and Z). The oblique cones show (except as to dimensions) to what portions of their surfaces the several portions of the surface of the oval body correspond. Thus the sides, A′, of the body correspond to the portion A of cone *x*, and the ends, B′, B′, correspond to the B portion cone Z. The correspondence will be more fully recognised as we proceed with the problem. The difference of obliquity between B′ and B is seeming only, not real; and arises simply from Z being turned round so that the whole of the *d* B *d d'* B′ *d'* (Fig. 52) of the cone shall be seen. If the representation of Z showed its full obliquity, then the line on it from base to apex would be the right-hand side line of the cone, and only half of the B portion could be seen.

FIG. 53



CASE I.—Patterns when the body is to be made up of four pieces.

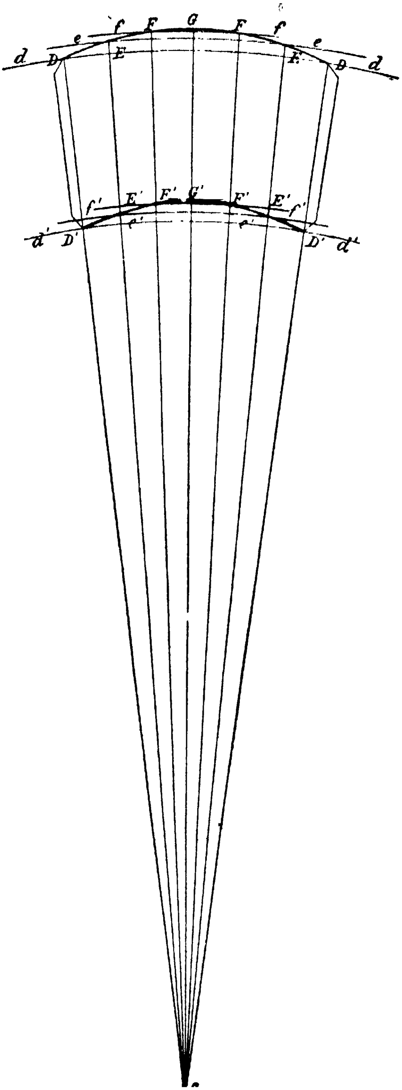
It is clear that we require two patterns; one for the two ends, and one for the two sides; also that the seams should correspond with the four lines $d\,d'$, where the portions of the respective frusta meet each other.

To draw the pattern for the ends.

Draw separately (Fig. 53) the $G'\,G\,f\,d\,d'$ portion of Fig. 52, thus. Draw any line X X, Fig. 53, and with any point O (corresponding to O, Fig. 52) in it as centre and O G (Fig. 52) as radius, describe an arc G d equal to G d of Fig. 52. Make G O′ equal to G O′ (Fig. 52), and with O′ as centre and O′ G′ (Fig. 52) as radius describe an arc G′ d' equal to G′ d' of Fig. 52. Joining $d\,d'$ completes the portion of Fig. 52 required. Now divide the arc G d into any number of equal parts, here three, in the points f and e. At G′ and O′ draw G′ G″, O′ O″ perpendicular to X X, and each equal to the given height of the body. Join O O″, G G″; produce them to their intersection in S (§ 80, p. 158); and from S let fall S S′ perpendicular to X X. Join O″ G″. With S′ as centre and S′ f, S′ e, and S′ d successively as radii, describe arcs cutting X X in F, E, and D. Join these points to S by lines cutting O″ G″ in F′, E′, and D′.

Next draw S G (Fig. 54) equal to S G (Fig. 53), and with S as centre and S F, S E, and S D (Fig. 53) successively as radii describe arcs $f\,f$, $e\,e$, and $d\,d$. With G as centre and radius equal to G f (Fig. 53) describe arcs cutting arc $f\,f$ right and left of S G in F and F. With each of these points, F and F as centre and same radius describe arcs cutting arc $e\,e$ right and left of S G in E and E. With same radius and each of the last-obtained points as centre describe arcs cutting $d\,d$ right and left of S G in D and D. Join all the points right and left of S G to S. With S as centre and S G″ (Fig. 53) as radius, describe an arc cutting S G in G′. With same centre and S F′ (Fig. 53) as radius, describe an arc $f'\,f'$ cutting the lines S F right and left of S G in F′ and F′. With same centre and S E′ (Fig. 53) as radius, describe an

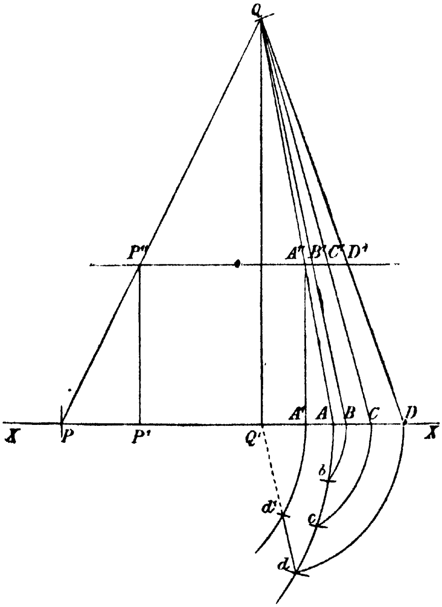
FIG. 54.



arc *e′ e′* cutting the lines S E right and left of S G in E′ and E′. Similarly by arc *d′ d′* obtain points D′ and D′. Through the points D, E, F, G, F, E, D, draw an unbroken curved line. Also through points D′, E′, F′, G′, F′, E′, D′, draw an unbroken curved line. Then D G D D′ G′ D′ will be the required pattern for the ends of the body. It is in fact the development of the B portion of cone Z of Plate I.

To draw the pattern for the sides.

FIG. 55.

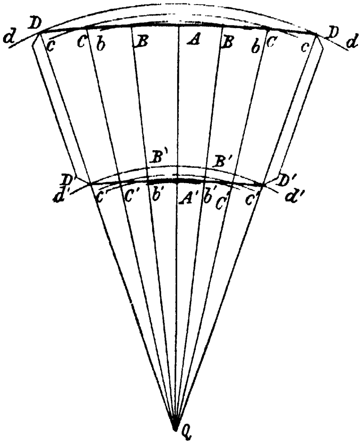


Draw separately (Fig. 55) the A′ A *b d d′* portion of Fig. 52, thus. Draw any line X X, Fig. 55, and with any point P (corresponding to P, Fig 52) in it as centre and P A (Fig. 52) as radius, describe an arc A *d* equal to A *d* of Fig. 52. Make

A P′ equal to A P′ (Fig. 52) and with P′ as centre and P′ A (Fig. 52) as radius describe an arc A′ *d′* equal to A′ *d′* of Fig. 52. Joining *d d′* completes the portion of Fig. 52 required. Now divide the arc A *d* into any number of equal parts, here three, in the points *b* and *c*. At A′ and P′ draw A′ A″, P′ P″ perpendicular to X X, and each equal to the given height of the body Join P P″, A A″, produce them to their intersection in Q (§ 80, p. 158); and from Q let fall Q Q′ perpendicular to X X. Draw a line through P″ A″. With Q′ as centre and Q′ *b*, Q′ *c*, and Q′ *d* successively as radii describe arcs cutting X X in B, C, and D. Join these points to Q by lines cutting that through P″ A″, in B′, C′, and D′.

Next draw Q A (Fig. 56) equal to Q A (Fig. 55), and with Q as centre and Q B, Q C, and Q D (Fig. 55) successively as

FIG. 56.



radii describe arcs *b b*, *c c*, and *d d*. With A as centre and radius equal to A *b* (Fig. 55) describe arcs cutting arc *b b* right and left of Q A in B and B. With each of these points

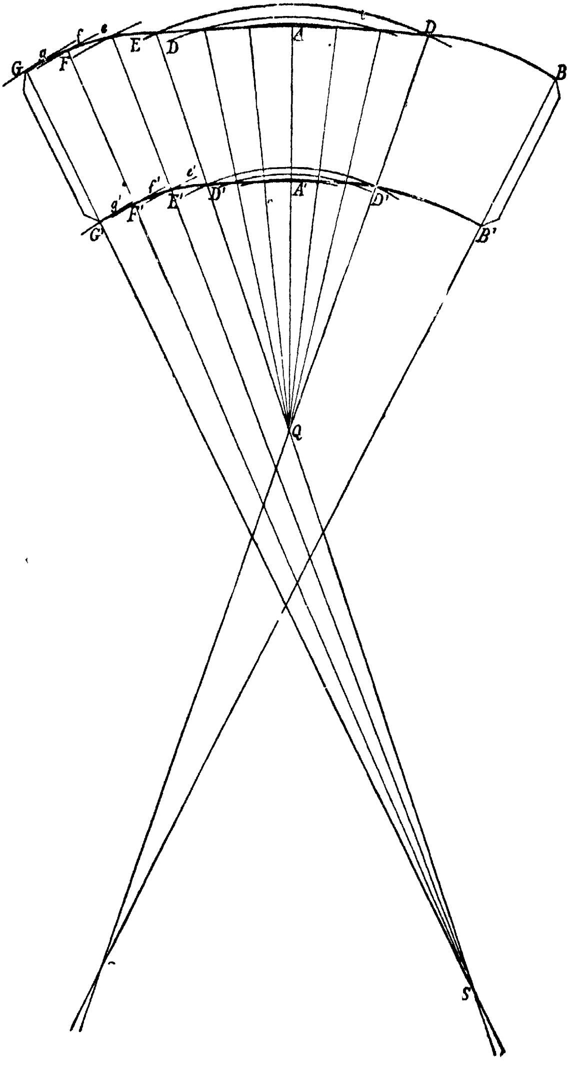
B and B as centre and same radius describe arcs cutting arc *c c* right and left of Q A in C and C. With same radius and each of the last-named points as centre describe arcs cutting arc *d d* right and left of Q A in D and D. Join all the points right and left of Q A to Q. With Q as centre and Q A″ (Fig. 55) as radius, describe an arc cutting Q A in A′. With same centre, and Q B′ (Fig. 55) as radius, describe an arc *b′ b* cutting the lines Q B right and left of Q A in B′ and B′. With same centre and Q C′ (Fig. 55) as radius, describe arc *c′ c′* cutting the lines Q C right and left of Q A in C′ and C′. Similarly by arc *d′ d′* obtain points D′ and D′. Through the points D, C, B, A, B, C, D, draw an unbroken curved line. Also through the points D′, C′, B′, A′, B′, C′, D′, draw an unbroken curved line. Then D A D D′ A′ D′ will be the required pattern for the sides of the body, and is in fact the development of the A portion of cone *x* of Plate I.

CASE II.—Pattern when the body is to be made up of two pieces.

In this case the seams are usually made to correspond with B B′ and G G′ (Fig. 52). It is evident that only one pattern is now required, made up of a pattern for the side A′ of the body (Plate I.) with right and left a half-end (B′, B′, Plate I.) pattern attached.

Draw (Fig. 57) a side pattern D A D D′ A′ D′ as described in Case I. Produce D Q and make D S equal to D S (Fig. 53). With S as centre and S E, S F, and S G (Fig. 53) successively as radii describe arcs *e*, *f*, and *g*, and with D as centre and *d e* (Fig. 53) as radius describe an arc cutting arc *e* in E. With same radius and E as centre describe an arc cutting arc *f* in F, and similarly with F as centre and same radius find G. Join E S, F S, G S. Now with S E′ (Fig. 53) as radius and S as centre describe an arc *e′* cutting S E in E′, and with S F′ and S G″ (Fig. 53) successively as radii describe arcs *f′* and *g′* cutting S F and S G in F′ and G′. From points D to G draw an unbroken curved line. Also from points D′ to G′ draw an unbroken curved line. Draw the other half-end

Fig 57.



pattern D B B′ D′ in the same manner; then G A B B′ A′ G′ will be the pattern required.

Case III.—Pattern when the body is to be made up of one piece.

We will put the seams at the middle of one end of the body, say, to correspond with B B′ (Fig. 52). We now need an end pattern (the end *d* G *d d′* G′ *d′* in plan), with side pattern attached right and left (*d* E *d d′* E′ *d′*, *d* A *d d′* A′ *d′* in plan), and attached to each of these a half-end pattern (*d* B B′ *d′*, *d* B B′ *d′* in plan). For want of space we do not give the pattern, but it is evident from what has just been stated, that the pattern will be double that shown in Fig. 57. It will be a useful exercise and should present no difficulties to the student, to himself draw the complete pattern, first drawing an end pattern (see Case I. and Fig. 54) and attaching right and left, a side pattern and a half-end pattern.

PROBLEM XXI.

To draw, **without long radii,** *the pattern for an oval unequal-tapering body, with top and base parallel (an oval bath, for instance). The height and the dimensions of the top and bottom of the body being given.*

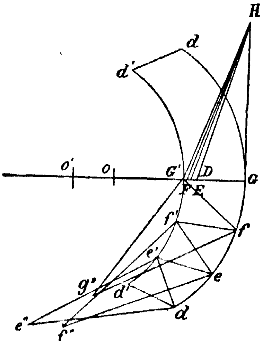
This problem is a fourth case of the preceding, and will be found very useful for both the end and side patterns, the radii of which are often of a most inconvenient length.

To draw the end pattern.

First draw (Fig. 58) the plan of the end of the body; that is the *d* G *d d′* G *d′* portion of Fig. 52. Divide the arcs G *d*, G′ *d′* each into the same number of equal parts, here three, in the points *f*, *e*, *f′*, *e′*; join *f f′*, *e e′*. Through G draw G H perpendicular to G G′ and equal to the given height of the body. From G along G G′ mark off G F, G E, and G D respectively equal to *f f′*, *e e′*, and *d d′*; join G′, F, E and D to H; then G′ H, F H, E H, and D H will be the true lengths of

G G′, *f f′*, *e e′*, and *d d′* respectively. Next join *f* G′; draw G′ *g″* perpendicular to *f* G′ and equal to the given height and join *f g″*; then *f g″* may be taken as the true length of G′ *f*. Similarly join *e f′*, *d e′*, and through *f′* and *e′* draw *f′ f″* and

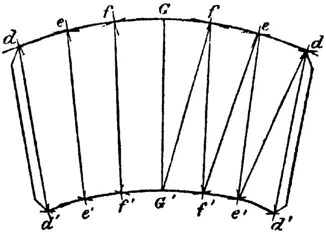
Fig. 58.



e′ e″ perpendicular to *f′ e* and *e′ d* respectively, and each equal to the given height, and join *e f″* and *d e″*; then *e f″* and *d e″* may be taken as the true lengths of *e f′* and *d e′* respectively.

Next draw (Fig. 59) a line G G′ equal to G′ H (Fig. 58) and with G′ and G as centres and radii respectively equal to *f g″* and G *f* (Fig. 58) describe arcs right and left of G G′, intersecting in *f* and *f*. With *f*, right of G, and G′ as centres, and radii respectively equal to F H and G′ *f′* (Fig. 58) describe arcs intersecting in *f′*, right of G′. With *f*, left of G, and G′ as centres, and radii respectively as before, describe arcs intersecting in *f′*, left of G′. With successively *f′* and *f*, right and left of G G′, as centres and radii respectively equal to *e f″* and *f e* (Fig. 58) describe arcs intersecting in *e* and *e*. With successively *e* and *f′* right and left of G′ G as centres and radii respectively equal to E H and *f′ e′* (Fig. 58) describe arcs intersecting in *e'* and *e'*; and with successively *e'* and *e* right and left of G G' as centres, and radii respectively equal to *d e''* and *e d* describe arcs intersecting in *d* and *d*. Also with successively *d* and *e'* as centres and D H and *e' d'* respectively as radii describe arcs intersecting in *d'* and *d'*.

FIG 59.



END PATTERN.

Through *d*, *e*, *f*, G *f*, *e*, *d*, draw an unbroken curved line. Also through *d'*, *e'*, *f'*, G', *f'*, *e'*, *d'*, draw an unbroken curved line. Join *d d'*, right and left of G G', then *d* G *d d'* G' *d'* will be the end pattern required.

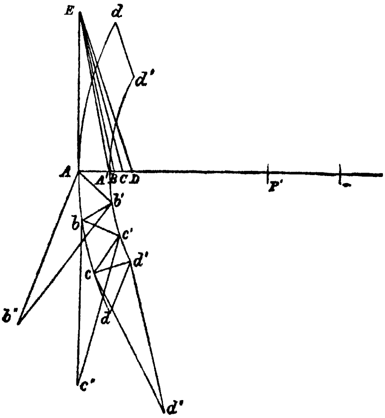
The lines *f f'*, *e e'*, G' *f*, *f' e*, &c., are not needed in the working, they are drawn for the reason stated in § 65, p. 121.

To draw the side pattern.

First draw (Fig. 60) the plan of the side of the body; that is the *d* A *d d'* A' *d'* portion of Fig. 52. Divide the arcs A *d*, A' *d'* each into the same number of equal parts, here three, in the points *b*, *c*, *b'*, *c'*; join *b b'*, *c c'*. Through A draw A E perpendicular to A A' and equal to the given height of the body. From A along A A' mark off A B, A C, and A D respectively equal to *b b'*, *c c'*, and *d d'*. Join A', B, C and D to E; then A' E, B E, C E, and D E will be the true lengths of A A', *b b'*, *c c'*, and *d d'* respectively. Next join *b'* A and draw *b' b''* perpendicular to *b'* A and equal to the given height.

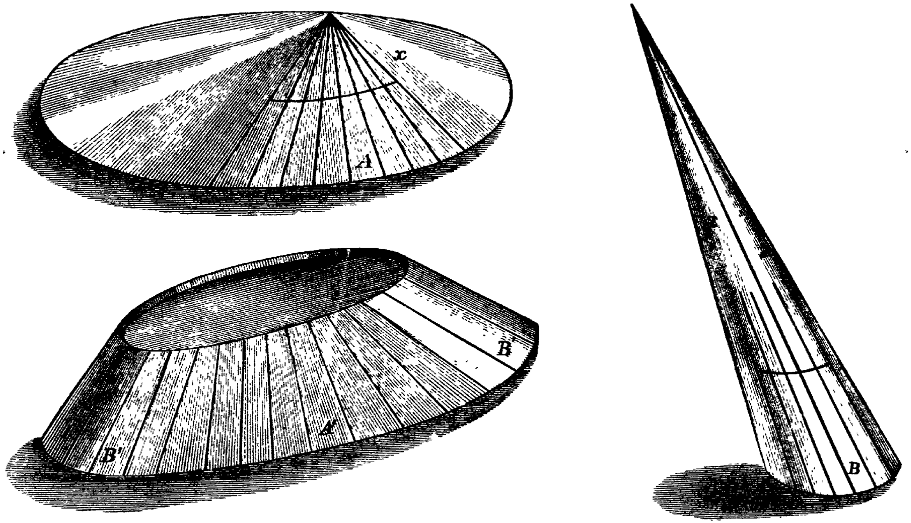
Join A *b*″; then A *b*″ may be taken as the true length A *b*′. Similarly join *c*′ *b* and *d*′ *c*; through *c*′ and *d*′ draw *c*′ *c*″, and *d*′ *d*″ perpendicular to *c*′ *b* and *d*′ *c* respectively, and each equal to the given height; join *b c*″, *c d*″, then *b c*″ and *c d*″ may be taken as the true lengths of *b c*′ and *c d*′ respectively.

FIG. 60



Next draw (Fig. 61) a line A A′ equal to A′ E (Fig. 60) and with A and A′ as centres and radii respectively equal to A *b*″ and A′ *b*′ (Fig. 60) describe arcs right and left of A A′, intersecting in *b*′ and *b*′. With *b*′, right of A′, and A as centres, and radii respectively equal to B E and A *b* (Fig. 60) describe arcs intersecting in *b*, right of A. With *b*′, left of A′, and A as centres, and radii respectively as before, describe arcs intersecting in *b* left of A. With successively *b* and *b*′ right and left of A A′ as centres, and radii respectively equal to *b c*″ and *b*′ *c*′ (Fig. 60) describe arcs intersecting in *c*′ and *c*′. With successively *c*′ and *b* right and left of A A′ as centres, and radii respectively equal to C E and *b c* (Fig. 60) describe arcs intersecting in *c* and *c*; and with successively *c* and *c*′ right

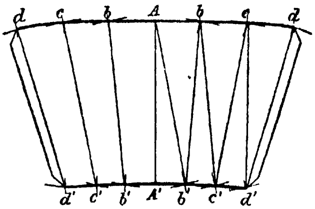
PLATE I. (see p. 169).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (*Part of Exhibit by the Author at the Inventions Exhibition,* 1885.)

and left of A A′ as centres, and radii respectively equal to $c\,d''$ and $c'd'$ (Fig. 60) describe arcs intersecting in d' and d'. Also with successively d' and c right and left of A A′ as centres, and D E and $c\,d$ respectively as radii describe arcs intersecting in d and d. Through d, c, b, A, b, c, d, draw an unbroken curved line. Also through d', c', b', A′, b', c', d', draw an unbroken curved line. Join $d\,d'$, right and left of A A′; then d A $d\,d'$ A′ d' will be the side pattern required.

FIG. 61.



SIDE PATTERN.

The remark about lines $f\,f'$, $e\,e'$, &c., in (end pattern) Fig. 59, applies to lines $c\,c'$, $b\,b'$, $b\,c'$, &c., in the present pattern.

PROBLEM XXII.

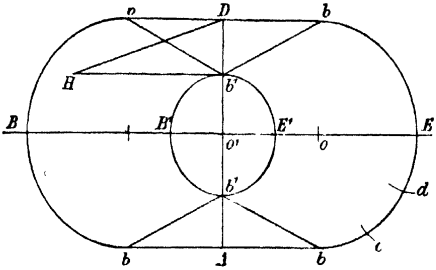
To draw the pattern for a tapering body with top and base parallel, and having circular top and oblong bottom with semicircular ends (tea-bottle top, for instance), the dimensions of the top and bottom of the body and its height being given.

Four cases will be treated of; three in this problem and one in the problem following (see also § 79, p. 157).

CASE I.—Pattern when the body is to be made up of four pieces.

Draw (Fig. 62) the plan of the body (see Problem VIII., p. 132) preserving of its construction the centres O, O′; the points b in plan of bottom where the extremities of the plan lines of the sides meet the extremities of the plan semicircles of the ends; and the points b' in plan of top where the sides and ends meet in plan. Join $b'\,b$ at the four corners. The ends $b\,B\,b\,b'\,B'\,b'$, and $b\,E\,b\,b'\,E'\,b'$ of the body (see *g*, p. 129), are portions of frusta of oblique cones. Making the body in four pieces it will be best that the seams shall correspond with the lines $A\,b'$, $B\,B'$, $D\,b'$, and $E\,E'$, then one pattern only, consisting of a half-end with a half-side pattern attached, will be required.

FIG. 62.

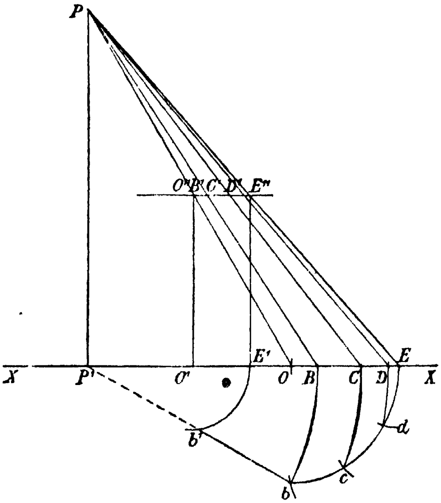


To draw the pattern.

Draw separately $E'\,E\,d\,b\,b'$ (Fig. 63), the $E'\,E\,d\,b\,b'$ portion of Fig. 62, thus. Draw any line X X and with any point O (to correspond with O, Fig. 62) in it as centre and O E (Fig. 62) as radius describe an arc (here a quadrant) $E\,d\,b$, equal to $E\,d\,b$ of Fig. 62. Make $E\,O'$ equal to $E\,O'$ (Fig. 62), and with O' as centre and $O'\,E'$ (Fig. 62) as radius describe an arc (here a quadrant) $E'\,b'$ equal to $E'\,b'$ of Fig. 62. Joining $b\,b'$ completes the portion of Fig. 62 required. Now divide $E\,b$ into any number of equal parts, here three, in the points *d* and *c*. From E' and O' draw $E'\,E''$, $O'\,O''$ perpendicular to X X and each equal to the given height of the body. Join $E\,E''$, $O\,O''$; produce them to intersect in P (§ 80, p. 158); from P let fall $P\,P'$ perpendicular to X X, and join $E''\,O''$. With P'

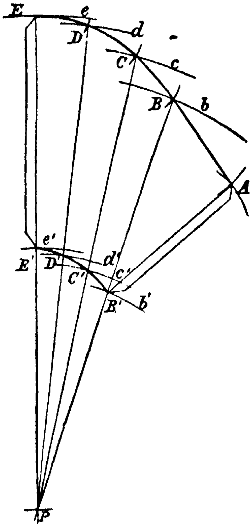
as centre and P′ *d*, P′ *c*, and P′ *b* successively as radii describe arcs cutting X X in D, C, and B. Join these points to P by lines cutting O″ E″ in D′, C′, and B′

FIG. 63.



Next draw (Fig. 64) a line P E equal to P E (Fig. 63), and with P as centre, and P D, P C, and P B (Fig. 63) successively as radii describe arcs *d*, *c*, and *b*. With E as centre and radius equal to E *d* (Fig. 63) describe an arc cutting arc *d* in D. With same radius and D as centre describe an arc cutting arc *c* in C, and with C as centre and same radius describe an arc cutting arc *b* in B. Join the points D, C and B to P. With P as centre and P E″ (Fig. 63) as radius describe an arc *e*′ cutting P E in E′. With same centre and P D′ (Fig. 63) as radius describe arc *d*′ cutting P D in D′. Similarly with same centre and P C′ and P B′ (Fig. 63) successively as radii find points C′ and B′. Through *b*′ (Fig. 62) draw *b*′ H perpendicular to *b*′ D and equal to the given height, and join D H, then D H will be the true length of the line of which *b*′ D is the plan, that is, will be the length of a slant of the body at the middle of the side, where one of the seams will come. With B′ and B (Fig. 64) as centres and radii respectively equal to D H and *b* A (Fig. 62) describe arcs intersecting in A. Through the points E, D, C, and B draw an unbroken curved line. Also through the points E′, D′, C′, and B′ draw an unbroken curved line. Join B A, A B′; then E C A B′ E′ will be the pattern required.

FIG. 64.

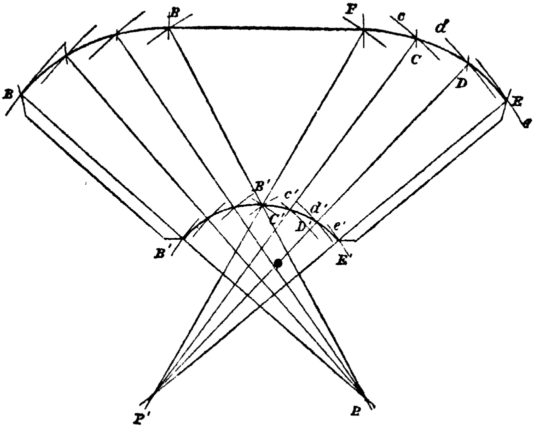


Case II.—Pattern when the body is to be made up of two pieces.

We will suppose the seams are to correspond with the lines B B′ and E E′. It is evident that here we need but one pattern only, which will combine a side of the body and two half-ends, in fact will be double that of Fig. 64.

First draw B B′ B′ B (Fig. 65) a half-end pattern exactly as the half-end pattern E E′ B′ B in Fig. 64 is drawn, and with B′ as centre and B′ B as radius and B as centre and *b b*

Fig. 65.



(Fig. 62) as radius, describe arcs intersecting in F. Join B F, F B′; and produce F B′ indefinitely. Make F P′ equal to P B (Fig. 63), and with P′ as centre and P C, P D and P E (Fig. 63) successively as radii describe arcs *c*, *d*, and *e*. With F as centre and *b c* (Fig. 63) as radius describe an arc cutting arc *c* in C, and with C as centre and same radius describe an arc cutting arc *d* in D. With same radius and D as centre describe an arc cutting arc *e* in E. Join C P′, D P′, E P′. Now with P′ as centre and P C′ (Fig. 63) as radius describe an arc *c′* cutting P′ C in C′, and with same centre and P D′, P E″ (Fig. 63) successively as radii describe arcs *d′* and *e′* respectively cutting P′ D and P′ E in D′ and E′. Through F, C, D, and E draw an unbroken curved line. Also through B′, C′, D′, and E′ draw an unbroken curved line. Then B B F E E′ B′ B′ will be the complete pattern required.

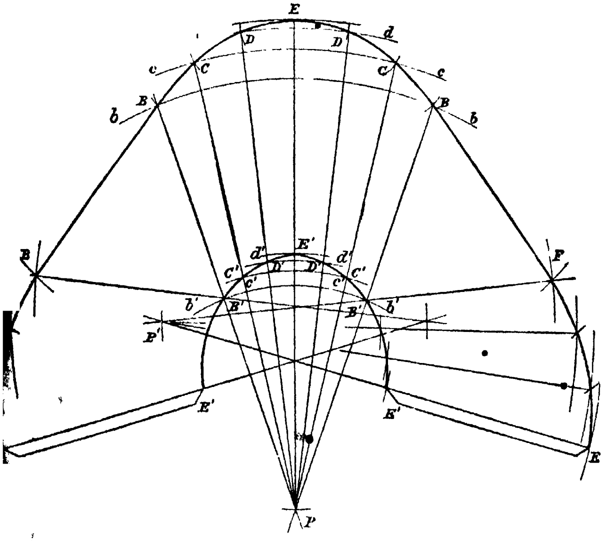
Case III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with B B′ (Fig. 62). We now need an end pattern (the end *b* E *b b′* E′ *b′* in plan), with right and left a side pattern attached (*b* A *b b′*, *b* D *b b′* in plan), and joined to each of these, a half-end pattern (*b b′* B′ B, *b b′* B′ B in plan).

First draw Fig. 63; then draw (Fig. 66) P E equal to P E (Fig. 63) and with P as centre and P D, P C and P B (Fig. 63) successively as radii describe arcs *d d*, *c c*, and *b b*. With E as centre and radius equal to E *d* (Fig. 63) describe arcs cutting arc *d d* right and left of P E in D and D. With points, D, D, successively as centres and same radius describe arcs cutting arc *c c* right and left of P E in C and C; and with same radius and the last found points as centres describe arcs cutting arc *b b* right and left of P E in B and B. Join all the points found to P. With P as centre and P E″ (Fig. 63) as radius describe an arc cutting P E in E′. With same centre and P D′ (Fig. 63) as radius describe an arc *d′ d′* cutting lines P D right and left of P E in D′ and D′. With same centre and P C′ (Fig. 63) as radius describe an arc *c′ c′* cutting lines P C right and left of P E in C′ and C′. Similarly by arc *b′ b′* find points B′ and B′. Through B, C, D, E, D, C, B, draw an unbroken curved line. Also through B′, C′, D′, E′, D′, C′, B′, draw an unbroken curved line. This gives us B E B B′ E′ B′ a complete end pattern. Now with B′ on the right-hand side of the end pattern as centre and B′ B as radius, and B as centre and *b b* (Fig. 62) as radius describe arcs intersecting in F. Join B F, F B′; produce F B′ indefinitely, and to F B′ attach the half-end pattern F E E′ B′ in precisely the same manner that

F E E′ B′ the half-end pattern in Fig. 65 is attached to the side pattern B F B′. By a repetition of the foregoing

Fig. 66.



onstruction on the left of the end pattern B E B B′ E′ B′ we an attach B B E E′ B′ and complete E E E E′ E′ E′ the pattern equired.

PROBLEM XXIII.

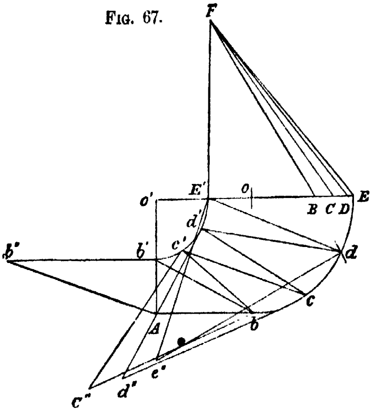
To draw, without long radii, *the pattern for a tapering body with top and base parallel, and having circular top and oblong bottom with semicircular ends. The dimensions of the top and bottom of the body and its height being given.*

This problem is a fourth case of the preceding, and is **xceedingly useful** where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces, as in Case I. of preceding problem).

(81.) Draw (Fig. 67) E' E *b* A *b'* one quarter of the plan of the body. Divide the quadrants E *b*, E' *b'*, each into the same number of equal parts, here three, in the points *d*, *c*, *d'*, *c'*; join *d d'*, *c c'*. Through E' draw E' F perpendicular to E' E and equal to the given height of the body. From E' along E' E mark off E' D, E C, and E' B respectively equal to *d d'*, *c c'*, and *b b'*; and join E F, D F, C F, and B F; then E F, D F, C F, and B F will be the true lengths of E E', *d d'*, *c c'*, and *b b'* respectively. Next join *d* E', and draw E' *e''* perpendicular to it and equal to the given height, and join *d e''*; then *d e''* may be taken as the true length of *d* E'. Also join *c d'* and *b c'*; through *d'* and *c'* draw *d' d''* and *c' c''* perpendicular to *c d'* and *b c'* respectively, and each equal to the given height, and join *c d''* and *b c''*; then *c d''* and *b c''* may be taken as the true lengths of *d' c* and *c' b* respectively. Through *b'* draw

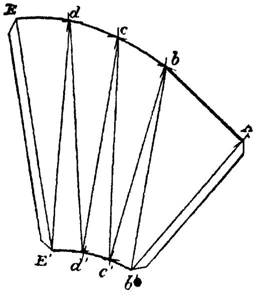
FIG. 67.



$b'b''$ perpendicular to $b'A$ and equal to the given height, and join Ab'', then Ab'' will be the true length of $b'A$.

Next draw (Fig. 68) EE' equal to EF (Fig. 67), and with E' and E as centres and radii respectively equal to de'' and Ed (Fig. 67) describe arcs intersecting in d, and with d and E' as centre and radii respectively equal to DF and $E'd'$ (Fig. 67) describe arcs intersecting in d'. With d' and d as centres and radii respectively equal to cd'' and dc (Fig. 67) describe arcs intersecting in c, and with c and d' as centres and radii respectively equal to CF and $d'c'$ (Fig. 67) describe arcs intersecting in c'. With c' and c as centres and radii respectively equal to bc'' and cb (Fig. 67) describe arcs intersecting in b. Similarly with b and c' as centres and radii respectively equal to BF and $c'b'$ (Fig. 67) describe arcs intersecting in b'. With b' and b as centres and radii respectively equal to $b''A$ and bA (Fig. 67) describe arcs intersecting in A. Through E, d, c, b, draw an unbroken curved line. Also through E', d', c', b', draw an unbroken curved line. Join $b'A$, bA; then $EcAb'c'E'$ is the pattern required.

FIG. 68.



(82.) The lines dd', cc', dE', &c., are drawn in Fig. 68 simply to show the position that the lines which correspond to them in Fig. 67 (dd', cc', dE', &c.) take upon the developed surface of the tapering body.

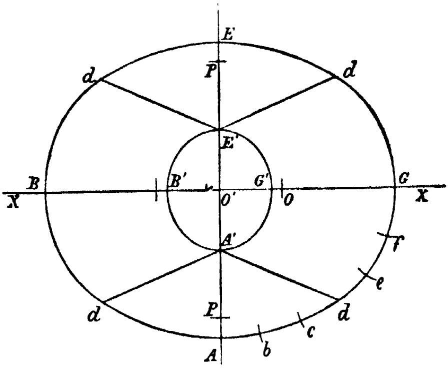
PROBLEM XXIV.

To draw the pattern for a tapering body with top and base parallel, and having an oval bottom and circular top (oval canister top, for instance). The height and dimensions of the top and bottom of the body being given.

Again four cases will be treated of; three in this problem and one in the problem following (see also § 79, p. 157).

Draw (Fig. 69) the plan of the body (see Problem IX., p. 132), preserving of its construction the centres O, O′, P, P, and the four points (*d*) where the end and side curves of the plan of the bottom meet one another, also the points A′ E′ where the axis A E cuts the circular top. Join *d* A′ (two places) and *d* E′ (two places). From the plan we know (see *g*, p. 120) that *d* G *d* A′ G′ E′, *d* B *d* A′ B′ E′, the ends of the body, are like portions of the frustum of an oblique cone; we also know that *d* A *d* A′, *d* E *d* E′, the sides of the body, are like portions of the frustum of an oblique cone.

FIG 69.



(83.) It is evident that in this problem the arcs d G d, E′ G′ A′ and d B d, E′ B′ A′ are, neither pair, proportional (§ 67, p. 124). We have hitherto in Problems XVIII., XX., and XXII. been dealing with proportional arcs. The working will therefore differ, though but slightly, from that of problems mentioned.

In Plate II. (p. 203) is a representation of the tapering body for which patterns are required, also of two oblique cones (x and Z). The oblique cones show to what portions of their surfaces the several portions of the tapering body correspond. Thus the sides, B′, of the body correspond to the B portion of cone Z, and the ends, A′, correspond to the portion A of cone x. The correspondence will be more fully recognised as we proceed with the problem.

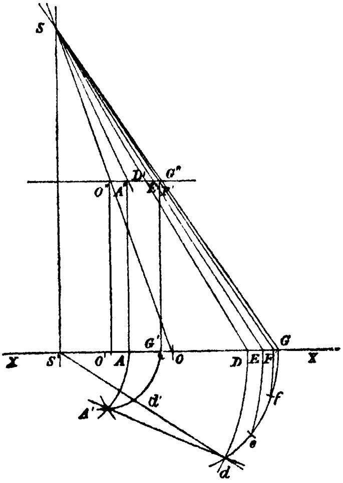
Case I.—Pattern when the body is to be made up of four pieces.

We will suppose the seams are to correspond with the plan lines G G′, B B′, A A′, E E′, of ends and sides, as in Problem XXII. Then one pattern only, consisting of a half-end pattern, with, attached, a half-side pattern will be required.

To draw the pattern.

Draw (Fig. 70) separately G′ G f d A′ the G′ G f d A′ portion of Fig. 69, thus. Draw any line X X and with any point O in it (corresponding to O, Fig. 69) as centre and O G (Fig. 69) as radius describe an arc G d, equal to G d of Fig. 69. Make G O′ equal to G O′ (Fig. 69), and with O′ as centre and O′ G′ (Fig. 69) as radius describe an arc G′ A′ equal to G′ A′ of Fig. 69. Joining d A′ completes the portion of Fig. 69 required Now divide G d into any number of equal parts, here three, in the points f and e. From G′ and O′ draw G′ G″, O′ O″ perpendicular to X X and each equal to the given height of the body. Join G G″, O O″; produce them to intersect in S (§ 80, p. 158); from S let fall S S′ perpendicular to X X, and join O″ G″. Now join d to S′, by a line cutting arc G′ A′ in d', then (§ 68, p. 124, and d, p. 126) *d′* and *d* are corresponding points and (§ 67, p. 124) the arcs G *d*, G′ *d′* are proportional; G G′ *d′ d* is the plan of a portion of an oblique-cone frustum, lying between the same generating lines, and *d d′* A′ is plan of a portion of the same frustum outside the generating line S′ *d*. With S′ as centre and S′ *f*, S′ *e*, and S′ *d* successively as radii describe arcs cutting X X in F, E, and D; join these points to S by lines cutting O″ G″ in F″, E′, and D′. With S′ as centre and radius the distance between S′ and A′ describe an arc A′ A cutting X X in A; from A draw A A″ perpendicular to X X and cutting O″ G″ in A″, and join A″ S.

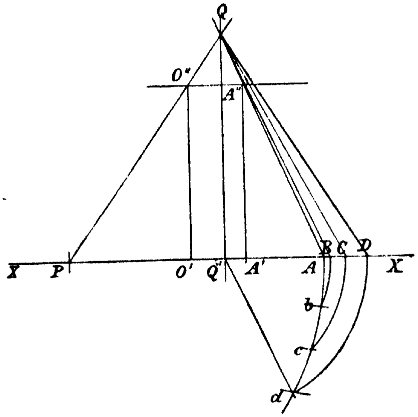
FIG. 70.



Next draw any line X X (Fig. 71) and with any point P in it (corresponding to P, Fig. 69) as centre and radius P A (the radius of arc *d* A *d*, Fig. 69) describe an arc A *d* equal to A *d* of Fig. 69. Make A A′ and A O′ respectively equal to

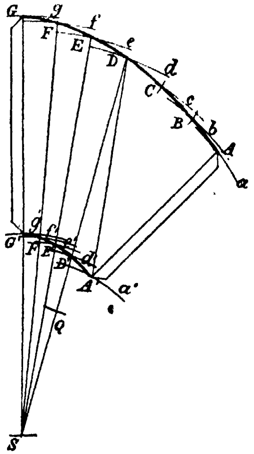
A A′ and A O′ (Fig. 69). Divide A *d* into any number of equal parts, here three, in the points *b* and *c*. From A and O′ draw A′ A″, O′ O″ perpendicular to X X and each equal to the given height of the body. Join A A″, P O″; produce them to intersect in Q (§ 80, p. 158); from Q let fall Q Q′ perpendicular to X X, and join O″ A″. With Q′ as centre and Q′ *b*, Q′ *c*, and Q′ *d* successively as radii describe arcs cutting X X in B, C, and D, and join these points to Q.

Fig. 71.



Next draw (Fig. 72) a line S G equal to S G (Fig. 70) and with S as centre and S F, S E, and S D (Fig. 70) successively as radii describe arcs *f*, *e*, and *d*. With G as centre and radius equal to G *f* (Fig. 70) describe an arc cutting arc *f* in F. With same radius and F as centre describe an arc cutting arc *e* in E, and with E as centre and same radius describe an arc cutting arc *d* in D. Join the points F, E, and D to S. With S as centre and S G″ (Fig. 70) as radius describe an arc *g*′ cutting S G in G′. With same centre and S F′ (Fig. 70) as radius describe arc f' cutting S F in F′. With same centre and S E′ (Fig. 70) as radius describe an arc e' cutting S E in E′, and with same centre and S D′ (Fig. 70) as radius describe arc d' cutting S D in D′. With same centre and S A″ (Fig. 70) as radius describe arc a', and with D′ as centre and radius d' A′ (Fig. 70) describe an arc intersecting arc a' in A′. Make D Q equal to D Q (Fig. 71) and with Q as centre and Q C,

FIG. 72.



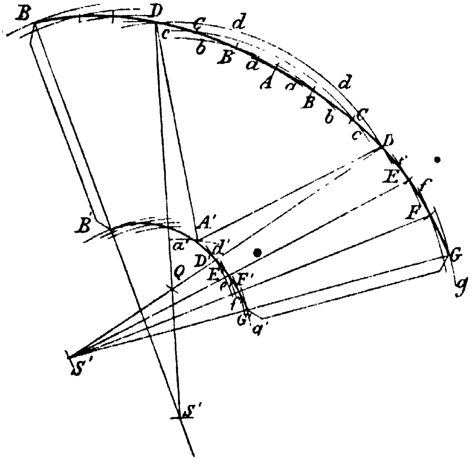
Q B, and Q A (Fig. 71) successively as radii describe arcs c, b, and a. With D as centre and $d\,c$ (Fig. 71) as radii describe an arc cutting arc c in C, and with C as centre and same radius describe an arc cutting arc b in B. Similarly with same radius and B as centre find point A. Join A A′. Through the points G, F, E, D, C, B, A draw an unbroken curved line. Also through the points G′, F′, E′, D′, A′ draw an unbroken curved line. Then G D A A′ E′ G′ is the pattern required.

CASE II.—Pattern when the body is to be made up of two pieces.

We will suppose the seams are to correspond with the lines G G′ and B B′. It is evident that here we need but one pattern only, which will combine a side of the body and two half-ends, in fact will be double that of Fig. 72.

First draw B D A′ B′ (Fig. 73) a half-end pattern exactly as the half-end pattern G D A′ G′ in Fig. 72 is drawn, and make D Q equal to D Q (Fig. 71). With Q as centre and

FIG. 73.



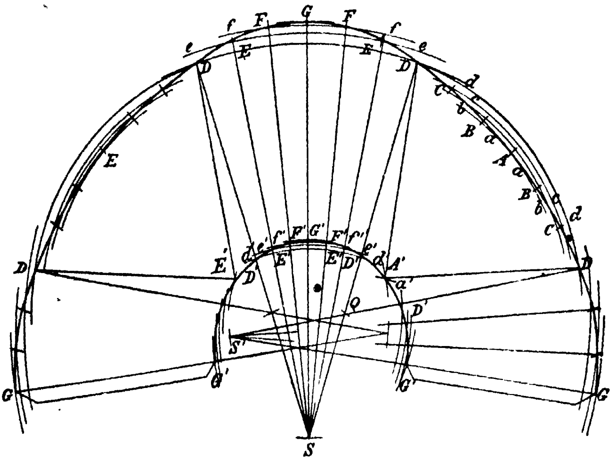
Q C, Q B, and Q A (Fig. 71) successively as radii describe arcs *d d*, *c c*, *b b*, and *a a*. With D as centre and radius equal to *d c* (Fig. 71) describe an arc cutting arc *c c* in C, and with same radius and C as centre describe arc cutting arc *b b* in B. With B as centre and same radius describe an arc cutting arc *a* in A, and with same radius and A as centre describe an arc cutting arc *b b* in B. Similarly with same radius and points B and C, to the right of A, successively as centres find points C and D. Join D Q and produce it indefinitely; make D S′ equal to S D (Fig. 70). With S′ as centre and S E, S F, and S G (Fig. 70) successively as radii describe arcs *e*, *f*, and *g*, and with D as centre and *d e* (Fig. 70) as radius describe an arc cutting arc *e* in E. With same radius and E as centre describe an arc cutting arc *f* in F. Similarly with F as centre and same radius find point G. Join the points E, F, and G to S′. With S′ as centre and S D′ (Fig. 70) describe arc *d*′ cutting S′ D in D′. With same centre and S E′ (Fig. 70) as radius describe arc *e*′ cutting S′ E in E′. Similarly with same centre and S F′ and S G″ (Fig. 70) successively as radii find points F′ and G′. Through the points D, C, B, A, B, C, D, E, F, G, draw an unbroken curved line. Also through points A′, D′, E′, F′, G′ draw an unbroken curved line. Then B A G G′ A B′ is the complete pattern required.

CASE III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with G G (Fig. 69). We now need an end pattern (the end *d* B *d* A B′ E′ in plan), with right and left a side pattern attached (*d* A *d* A′, *d* E *d* E′ in plan), and joined to each of these a half end pattern (*d* A′ G′ G, *d* E′ G′ G in plan).

First draw Figs. 70 and 71; then draw (Fig. 74) S G equal to S G (Fig. 70), and with S as centre and S F, S E, and S D (Fig. 70) successively as radii describe arcs *f f*, *e e*, and ***d d***. With G as centre and radius G *f* describe arcs cutting arc *f f* right and left of S G in F and F. With points F, F, successively as centres and same radius describe arcs cutting arc ***e e*** right and left of S G in E and E, and with same radius, and the last found points as centres describe arcs cutting arc *d d* right and left of S G in D and D. Join all the points found to S. With S as centre and S G″ (Fig. 70) as radius describe an arc cutting S G in G′. With same centre and S F′ (Fig. 70) as radius describe an arc *f′ f′* cutting lines S F right and left of S G in F′ and F′. With same centre and S E′ (Fig. 70) describe an arc *e′ e′* cutting lines S E right and left of S G in E′ and E′, and with same centre and S D′ (Fig. 70) as radius describe arc *d′ d′* cutting lines S D right and left of S G in D′ and D′. With S as centre and D′ right and left of S G as centres and radii respectively equal to S A″ and *d′* A′ (Fig. 70) describe arcs intersecting in A′ and E′. Through D, E, F, G, F, E, D, draw an unbroken curved line. Also through E′, D′, E′, F′, G′, F″, E′, D′, A′, draw

FIG. 74.



an unbroken curved line, and join D E′, D A′. This gives us D G D A′ G′ E′ a complete end pattern. Now attach the side pattern D A D A′ and the half-end pattern D G G′ A′ to the right and left of the complete end pattern we started with, in precisely the same manner that the side pattern D A D A′ and half-end pattern D G G′ A′ in Fig. 73 is attached to D A′, which corresponds to D A′ in Fig. 74. This will complete G E G A G G′ G′ G′ the pattern required.

PROBLEM XXV.

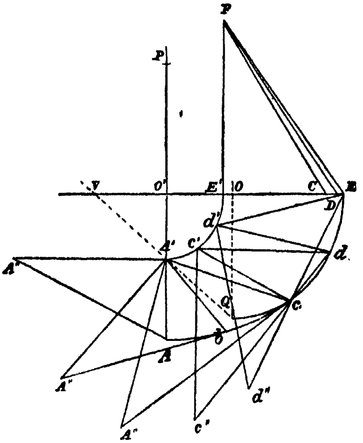
To draw, **without long radii,** *the pattern for a tapering body with top and base parallel, and having an oval bottom and circular top. The height and dimensions of the top and base of the body being given.*

This problem is a fourth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces, as in Case I. of preceding problem).

(84.) Draw (Fig. 75) E *d b* A A′ *d*′ E′ one quarter of the plan of the body. Join *c* (the point where the end and side curves of the plan of the bottom meet) to A′, the extremity of the quadrant E A′. We must now get at corresponding points in

Fig. 75.



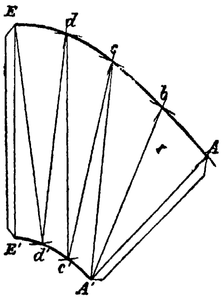
the arcs E *c*, E' A'. To do this, as the arcs are not proportional (§ 67), we must find the plan of the apex of the oblique cone of a portion of which E *c* A' E' is the plan. It is in the finding of these points that our present working differs from the working of Problems XIX., XXI., XXIII., and XXVII., where the corresponding points *d d'*, *c c'* (Figs. 50, 58, 60, and 82) are found. With radius O E produce arc E *c* indefinitely, and through O draw O Q perpendicular to E' E and cutting E *c* produced in Q. Then E Q is a quadrant, and E Q, E' A' (each a quadrant) are proportional. In Q and A' therefore we have corresponding points (§ 68), as well as in E and E', which are points on the longest generating line (*b*, p. 126); and the intersection of O O' produced, of which E E' is part, and Q A' joined and produced will give us V, the required plan of the apex. Next divide E *c* into any number of equal parts, here two, in point *d*. Join *d* V, *c* V cutting E' A' in *d'* and *c'*, respectively (the lines from *d* and *c* are not carried to V in the fig.), then *d* and *d'*, *c* and *c'* are corresponding points.

Next through E' draw E' F perpendicular to E' E and equal to the given height; from E' along E' E mark off E' D, E' C, respectively equal to *d d'* and *c c'*, and join E F, D F, and C F; then E F, D F and C F will be the true lengths of E E', *d d'*, and *c c'*. Join E *d'* and *d c'*; through *d'* and *c'* draw *d' d''* and *c' c''* perpendicular to *d'* E and *c' d* respectively, and each equal to the given height, and join E *d''*, *d c''*; then E *d''* and *d c''* may be taken respectively as the true lengths of *d'* E and *c' d*. Now divide arc A *c* into any number of equal parts, here two, in the point *b*, and join *b* A'; through A' draw three lines A' A''; one perpendicular to A' *c*, the second perpendicular to A' *b*, and the third perpendicular to A' A, and each equal to the given height, and join *c* A'', *b* A'', and A A''; then *c* A'', *b* A'', and A A'' may be taken as the true lengths of A' *c*, A' *b*, and A' A respectively.

Next draw (Fig. 76) E E' equal to E F (Fig. 75), and with E and E' as centres and radii respectively equal to E *d''* and E' *d''* (Fig. 75, describe arcs intersecting in *d'*, and with *d'* and E as centres and radii respectfully equal to D F and E *d*

(Fig. 75) describe arcs intersecting in *d*. With *d* and *d'* as centres and radii respectively equal to *d c''* and *d' c'* (Fig. 75) describe arcs intersecting in *c'*, and with *c'* and *d* as centres and radii respectively equal to C F and *d c* (Fig. 75) describe arcs intersecting in *c*. With *c* and *c'* as centres and radii respectively equal to *c* A'' and *c'* A' (Fig. 75) describe

Fig. 76.

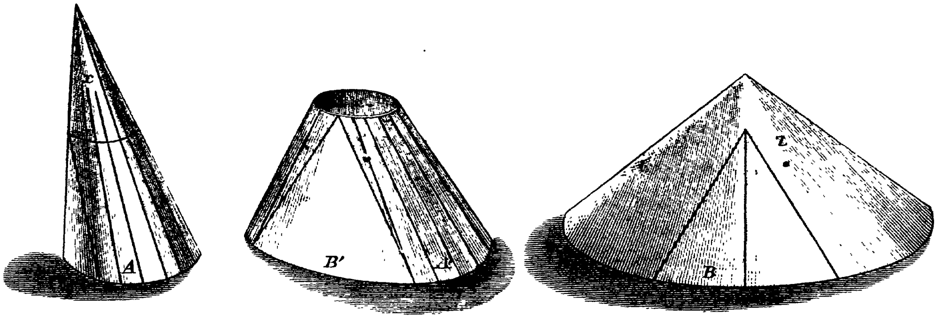


arcs intersecting in A', and with A' and *c* as centres and radii *b* A'' and *c b* (Fig. 75) describe arcs intersecting in *b*. Similarly with A' and *b* as centres and radii respectively equal to A A'' and *b* A (Fig. 75) describe arcs intersecting in A. Join A A'. Through E, *d*, *c*, *b*, A draw an unbroken curved line. Also through E', *d'*, *c'*, A' draw an unbroken curved line. Then E *c* A A' E' is the pattern required.

The lines *d d'*, *c c'*, E *d'*, *d c'*, &c., are not needed for the working, they are drawn for the reason stated in § 82, end of Problem XXIII.

(85.) If V is inaccessible, corresponding points *c*, *c'*, *d*, *d'* can thus be found. From the point E' along the arc E' A' set off an arc proportional to the arc E *c* in the following manner. Join O *c* (line not shown in fig.) and through O' draw O' *c'* (also not shown in fig.) parallel to O *c* and cutting arc E' A' in *c'*; then arcs E' *c'* and E *c* will be proportional. (The student must particularly notice this

PLATE II. (see p. 193).



Drawings from Models in South Kensington Museum. (*Part of Exhibit by the Author at the Inventions Exhibition*, 1885.)

method of drawing proportional arcs. It is outside the scope of the book to prove the method.) Now divide arcs E′ c′, E c each into the same number of equal parts, here two, in the points *d* and *d′*; then *d*, *d′* and *c*, *c′* are corresponding points.

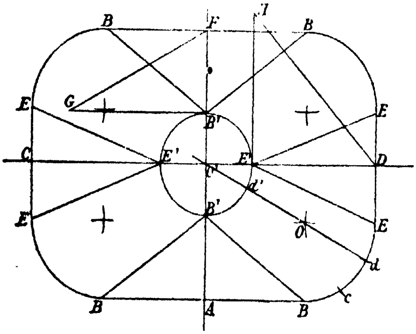
PROBLEM XXVI.

To draw the pattern for a tapering body with top and base parallel, and having oblong bottom with round (quadrant) corners, and circular top. The dimensions of the top and base of the body and its height being given.

Again four cases will be treated of, three in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 77) the plan of the body (see Problem X., p. 133) preserving of its construction the centres O O′ and the points E′ B′ where the flat sides and flat ends meet the circle of the top. Join B B′, E E′ each in four places. From the plan we know (see *g*, p. 129) that the round corners of the body are portions of frusta of oblique cones.

FIG. 77.



(86.) Looking at the plan, we can at once see that what we have to deal with differs somewhat from what has as yet been before us. Hitherto a line passing through the centres of the plan arcs bisected the arcs, and the cone development was consequently identical each side of a central line. In Fig. 77, however, the line drawn through O O′ does not bisect the plan arcs E B, E′ B′. This affects the working but little, as will be seen.

In Plate III. (p. 213) the tapering body is represented; also an oblique cone Z, the A portion of which corresponds to the A′ portion of the body, and the development of the former is the development of the latter.

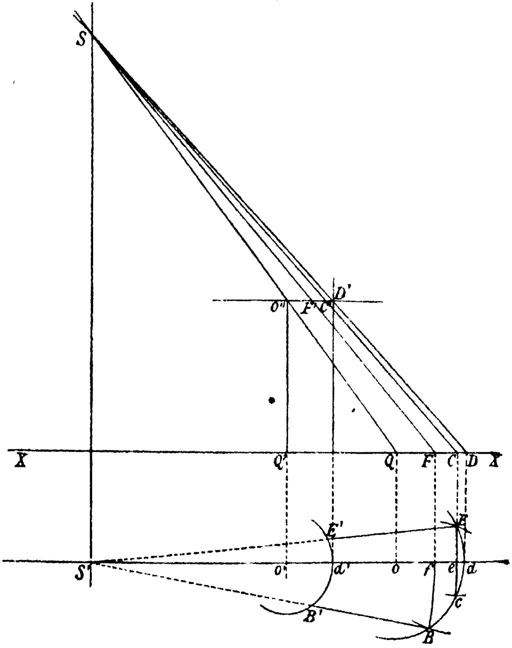
Case I.—Pattern when the body is to be made up of four pieces.

We will suppose the seams to correspond with the plan lines C E′, D E′, F B′, A B′, of ends and sides, as in Problems XXII. and XXIV. just preceding. Then one pattern, comprising a half-end, a complete corner, and a half-side, will be the pattern required.

To draw the pattern.

Draw separately (Fig. 78) an E E′, B′ B portion of Fig. 77, thus. Draw an indefinite line S′ *d* (Fig. 78), and with any point O (corresponding to O, Fig. 77) in it as centre and O B (Fig. 77) as radius describe an arc B E. Join O O′ (Fig. 77) and produce it cutting arc B E in *d*; make *d* B and *d* E (Fig. 78) equal respectively to *d* B and *d* E (Fig. 77). Now (Fig. 78) make *d* O′ equal to *d* O′ (Fig. 77), and with O′ as centre and O′ B′ (Fig. 77) as radius describe an arc B′ E′. Make *d′* B′ and *d′* E′ equal respectively to *d′* B′ and *d′* E′ (Fig. 77). Joining E E′, B B′ completes the portion of Fig. 77 required. Now divide (Fig. 77) B E into any number of parts. It is convenient to take *d* as one of the division points, and to make *d c* equal to *d* E; leaving *c* B without further division, thus making the division of B E into three portions not all equal. In actual practice the dimensions of the work will suggest the number of parts expedient. Now (Fig. 78) make *d c* equal to *d c* (Fig. 77), then B E (Fig. 78) will be divided correspondingly to B E (Fig. 77). Draw X X parallel to S′*d*; and at *d* and O draw *d* D, O Q perpendicular to S′*d*; and meeting X X in D and Q; also through *d*′ and O′ draw *d*′ D′,

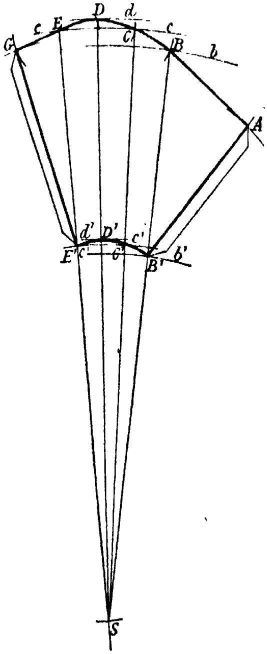
Fig. 78.



O′ O″ perpendicular to S′*d*; the line O′ O″ cutting X X in Q′. Make Q′ O″ equal to the given height of the body, and draw O″D′ parallel to X X. Join D D′, Q O″; produce them to intersect in S (§ 80, p. 158); and from S let fall a perpendicular to S′*d*, cutting S′*d* in S′. With S′ as centre and S′ E, or S′ *c* (which is equal to S E) and S′ B successively as radii describe arcs cutting S′ *d* in *e* and *f*. Draw *e* C, *f* F perpendicular to X X and cutting it in C and F; also join C S, F S, cutting O″ D′ in C′ and F′.

Next draw S D (Fig. 79) equal to S D (Fig. 78), and with S as centre and S C, S F′ (Fig. 78) successively as radii

FIG. 79.



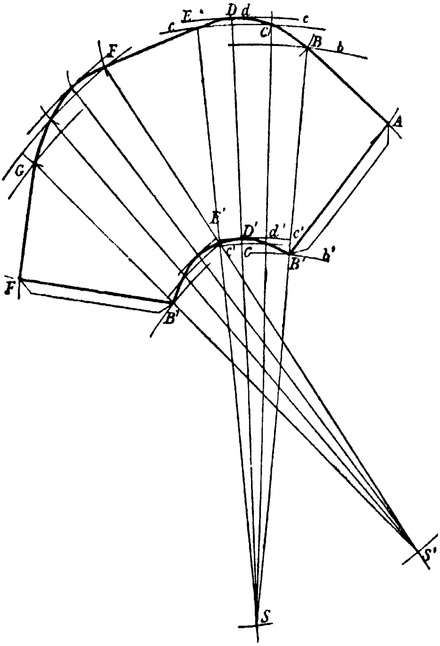
describe arcs *c* and *b*. With D as centre and radius equal to *d* E (Fig. 78) describe arcs cutting arc *c c* in E and C. With C as centre and radius *c* B (Fig. 78) describe an arc cutting arc *b* in B. Join E, C, and B to S. Make S D′ equal to S D′ (Fig. 78) and with S as centre and S C′ (Fig. 78) as radius describe an arc *c′ c′* cutting S E and S C in E′ and C′ respectively. With same centre and S F′ (Fig. 78) as radius describe an arc *b′* cutting S B in B′. Through E, D, C, and B draw an unbroken curved line. Also through E′, D′, C′, and B′ draw an unbroken curved line; this will complete the pattern of the round corner. To attach the half-end and half-side patterns to E E′ and B B′ respectively, the true lengths of E′ D and B′ A (Fig. 77) must be found. Draw (Fig. 77) E′ H perpendicular to E′ D and equal to the given height of the body; join D H, then D H is the true length of E′ D. The lines B′ E and B′ A being equal, their true lengths are equal, we will therefore for convenience find the true length of B′ A in that of B′ F. Draw B′ G perpendicular to B′ F and equal to the given height, join F G, then F G is the true length required. Now with E′ (Fig. 79) as centre and D H (Fig. 77) as radius, and E as centre and radius E D (Fig. 77) describe arcs intersecting in G. Join E G, G E′; this attaches to E′ E the half-end pattern. With B′ (Fig. 79) as centre and F G (Fig. 77) as radius, and B as centre and radius B A (Fig. 77) describe arcs intersecting in A. Join B A, A B′; this attaches to B′ B the half-side pattern. Then A B′ E′ G is the complete pattern required.

CASE II.—Pattern when the body is to be made up of two pieces.

Here it will be best that the seams shall correspond with the lines A B′, F B′, that is with the middle of each side. The required pattern will then be double that of Case I.

Draw (Fig. 80) E B B′ E′, the corner pattern, in exactly the same manner that E B B′ E′ (Fig. 79) is drawn. With E′ as centre and E′ E as radius, and E as centre and E E (Fig. 77) as radius describe arcs intersecting in F. Join E F, F E′. Produce F E′ indefinitely and make F S′ equal to E S. Using S′ as centre, the round corner F G B′ E′ can be drawn as was E B B′ E. With B′ as centre and F G (Fig. 77) as radius, and B as centre and B A (Fig. 77) as radius describe arcs intersecting in A. Similarly with B′ and G as

Fig. 80.

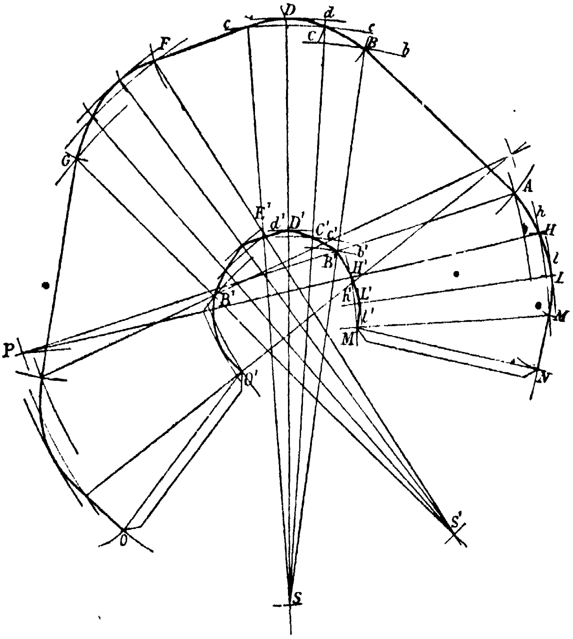


centres and same radii respectively describe arcs intersecting in F. Join B A, A B′, G F, F B′, then A D F F B′ E′ B′ will be the pattern required.

Case III.—Pattern when the body is to be made up of one piece.

We will suppose the seam to correspond with C E′ the middle of one end. Draw G F D B B′ E′ B′ (Fig. 81) in the same manner that G F D B B′ E′ B′ (Fig. 80) is drawn. With B′ as centre and B′ B as radius, and B as centre and B B (Fig. 77) as radius describe arcs intersecting in A. Join B A, A B′; produce A B′ indefinitely. Make A P equal to S F (Fig. 78). With P as centre and S C and S D succes-

FIG. 81.



sively as radii describe arcs *h* and *l*, and with A as centre and B *c* (Fig. 78) as radius describe an arc cutting arc *h* in H. With H as centre and *c d* (Fig. 78) as radius describe an arc cutting arc *l* in L; and with L as centre and same radius describe an arc cutting arc h in M. Join H, L, and M to P. With P as centre and S C′ (Fig. 78) as radius describe arc h' cutting P H and P M in H′ and M′ respectively. With same centre and S D′ (Fig. 78) as radius describe arc l' cutting P L in L′. Through the points A, H, L, M draw an unbroken curved line. Also through the points B′, H′, L′, M′ draw an unbroken curved line. With M′ as centre and D H (Fig. 77) as radius, and M as centre and E C (Fig. 77) as radius describe arcs intersecting in N. Join M N, N M′. Repeating the working to the left of B′G, the G O O′B′ portion of the pattern can be drawn which completes the pattern required.

PROBLEM XXVII.

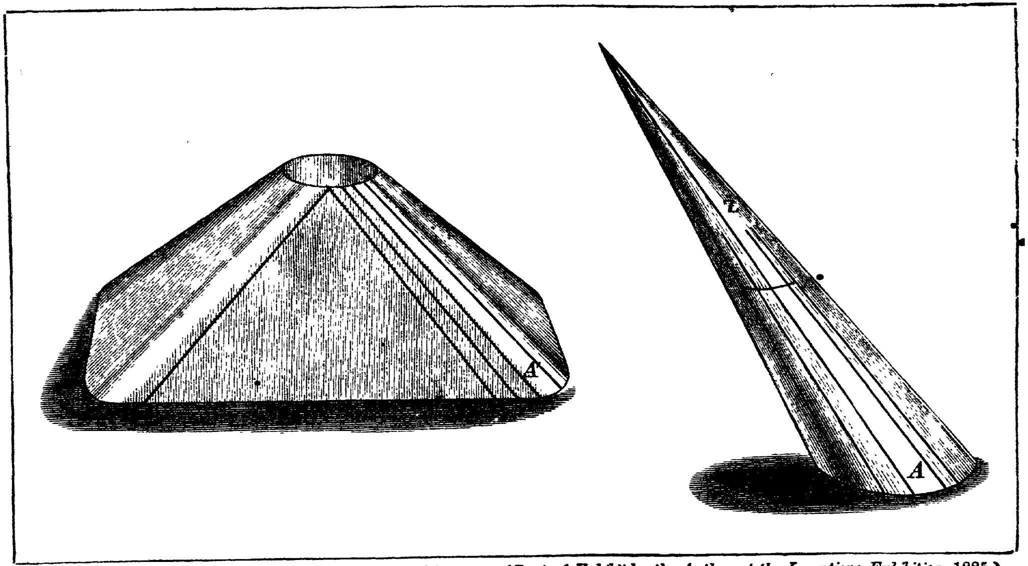
To draw, without long radii, *the pattern for a tapering body with top and base parallel, and having oblong bottom with round quadrant corners, and circular top. The dimensions of the top and base of the body and its height being given.*

This problem is a fourth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces as in Case I. of preceding problem).

Draw (Fig. 82) E c b A b' c' d', one quarter of the plan of the body. Divide the arc (quadrant) $d\,b$ into any number of equal parts, here two, in the point c, and the arc $d'\,b'$ into the same number of equal parts in the point c'; and join $c\,c'$. Through d' draw d' F perpendicular to d' E and equal to the given height of the body. From d' along d' E mark off d' B, equal to $b\,b'$, and d' D equal to $c\,c'$ and $d\,d'$ (which two lines have happened to come in this particular fig. so nearly equal that we may take them as equal), and join B F, E F, and D F; then B F and E F will be the true lengths of $b\,b'$ and E d' respectively, and D F may be taken as the true

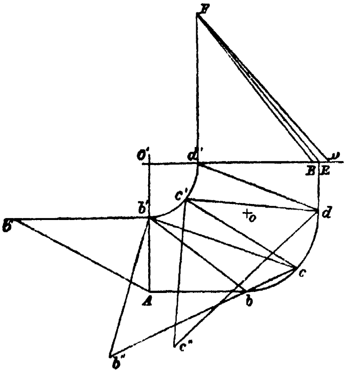
PLATE III. (see p. 206).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (*Part of Exhibit by the Author at the Inventions Exhibition*, 1885.)

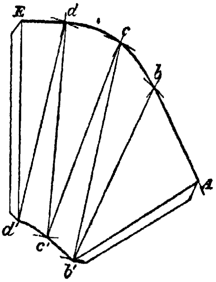
length of both *c c′* and *d d′*. Next join *c′ d*, *b′ c*; draw *c′ c″*, *b′ b″* perpendicular to *c′ d*, *b′ c* respectively, and each equal to the given height; and join *d c″* and *c b″*; then *d c″*, *c b″* may be taken as the true lengths of *c′ d* and *b′ c* respectively. Draw *b′ b″* perpendicular to *b′* A and equal to the given height, and join A *b″*, then A *b″* is the true length of *b′* A.

Fig. 82.



Next draw (Fig. 83) *d d′* equal to D F (Fig. 82), and with *d* and *d′* as centres and radii respectively equal to *d c″* and *d′ c′* (Fig. 82), describe arcs intersecting in *c′*. With *c′* and *d* as centres, and radii respectively equal to D F and *d c* (Fig. 82) describe arcs intersecting in *c*; and with *c* and *c′* as centres and radii respectively equal *c b″* and *c′ b′* (Fig. 82) describe arcs intersecting in *b′*; also with *b′* and *c* as centres and radii respectively equal to B F and *c b* (Fig. 82) describe arcs intersecting in *b*. With *d′* and *d* as centres and radii respectively equal to E F and *d* E (Fig. 82) describe arcs intersecting in E; and with *b′* and *b* as centres and radii respectively equal to *b″* A and *b* A (Fig. 82) describe arcs intersecting in A. Through d, c, b, draw an unbroken curved line. Also through d', c', b' draw an unbroken curved line.

FIG. 83.



Join dE, d'E, bA, and b'A; then EcA$b'd'$ is the pattern required.

The lines cc', bb', &c., are not needed for the working, they are drawn for the reason stated in § 82, end of Problem XXIII.

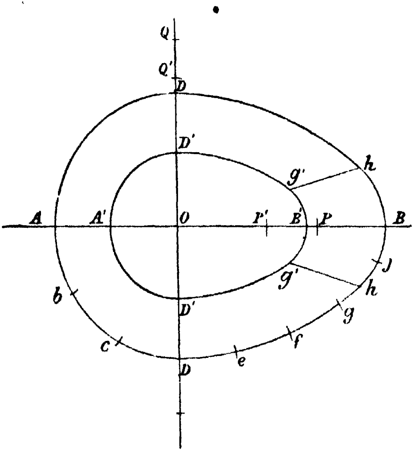
PROBLEM XXVIII.

To draw the pattern for an Oxford hip-bath, the like dimensions to those for Problem XI. being given.

It is only necessary to treat of two cases, one in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 84) the plan of the body (see Problem XI., p. 134), preserving of its construction the centres O, P′, P, Q′, Q; the points D, D′ (two sets) in which the arcs, in plan, of the back and sides meet each other; and the points h, g' (two sets) in which the plan arcs of the sides and front meet each other. Join hg' (two places) as shown in the fig. Examining the plan of the bath we see (d, p. 55) that the back of it, D A D D′ A′ D′, is a portion of a right cone; that the sides D D′ *g′ h* are (*g*, p. 120) each of them a portion of an oblique cone; and that the portion *h g′ g′ h* is also a portion of an oblique cone. Similarly as in Problem XXIV. (§ 83, p. 193) the arcs D *h*, D′ *g′* and B *h*, B′ *g′* are, neither pair, proportional.

Fig. 84.



In Plate IV. (p. 227) is a representation of an Oxford hip-bath, also of a right cone *x*, and two oblique cones Y and Z. The cones show to what portions of their surfaces the several portions of the bath correspond. Thus the back, A′, of the bath corresponds to the A portion of right cone *x*; the half-fronts, C′, of the bath correspond each of them to the C portion of oblique cone Y, and the sides, B′, of the bath correspond each of them to the B portion of oblique cone Z.

Patterns when the body is to be made up of three pieces.

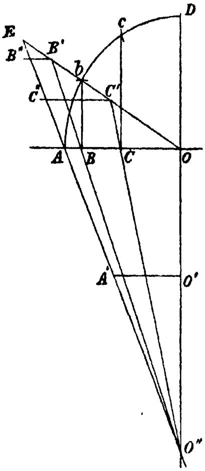
We will put the seams to correspond with the lines D D′

(two places), and B B′. Clearly, only two patterns will be required, one for the back of the bath, and the other for a complete side and a half-front.

To draw the pattern for the back.

Draw E A A′ O′ O (Fig. 85) the elevation of the back (see

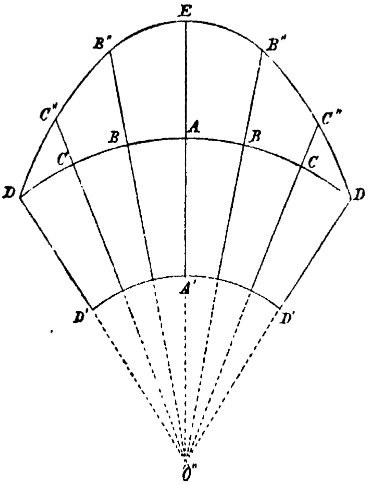
FIG. 85.



Problem XI. and Fig. 24*a*, p. 135) and produce A A′, O O′ to intersect in O″. With O as centre and O A as radius describe a quadrant A D (corresponding with A D, Fig. 84), and divide it into any number of equal parts, here three, in the points *b* and *c*. Draw *c* C, *b* B perpendicular each of them to A O and cutting it in points B and C. Join O″ C, and produce it to cut O E in C′; and through C′ draw C′ C″ parallel to A O and cutting O′ E in C″. Join O″ B and produce it to cut O E in B′; and through B′ draw B′ B″ parallel to A O and cutting O″ E in B″. Then A C″ is the true length of C C′, and A B″ the true length of B B′

Next draw (Fig. 86) O″A equal to O″A (Fig. 85), and with O″ as centre and radius O″A describe an arc D A D, and with same centre and O″A′ (Fig. 85) as radius describe an arc D′A′D′. Mark off, right and left of A, A B, B C, and C D, each equal to A *b* (Fig. 85), one of the equal parts into which the quadrant A D is divided. Join D O″, C O″, B O″ right and left of A O″, and produce O″A, O″B, O″C indefinitely. Make A E equal to A E (Fig. 85); B B″ equal to A B″ (Fig. 85); and C C″ equal to A C″ (Fig. 85); and through D, C″, B″, E, B″, C″, D, draw an unbroken curved line. Then D E D D′ A′ D′ is the required pattern for back of bath.

FIG. 86.



To draw the pattern for a side and a half-front.

Draw separately D′ D *f h g*′ (Fig. 87), the D′ D *f h g*′ portion of Fig. 84, thus. Draw any line X X and with any point Q (to correspond with Q, Fig. 84) in it as centre and (same

FIG. 87.

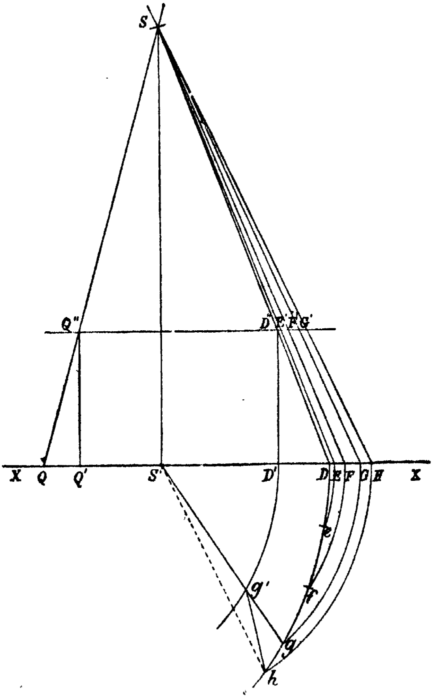
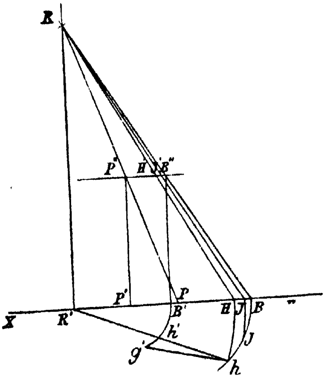


fig.) Q D (the D on the further side of A B) as radius describe an arc D *h* equal to D *h* of Fig. 84. Make D Q′ equal to D Q′ (Fig. 84), and with Q′ as centre and Q′ D′ (the further D′, Fig. 84) as radius describe an arc D′ *g*′ equal to D′ *g*′ of Fig. 84.

Joining *h g'* completes the portion of Fig. 84 required. Now from D' and Q' draw D' D'', Q' Q'' perpendicular to X X and each equal to the given height of the D B D portion of the bath. Join D D'', Q Q''; produce them to intersect in S (§ 80, p. 158); and from S let fall S S' perpendicular to X X. Join S' *g'*, and produce it to cut arc D *h* in *g*, then *g* and *g'* will be corresponding points. Divide D *g* into any number of equal parts, here three, in the points *e* and *f*. Join Q'' D''. With S' as centre and S' *e*, S' *f*, and S' *g* successively as radii describe arcs cutting X X in E, F, and G; join these points to S by lines cutting Q'' D'', produced, in E', F', and G'. With S' as centre and S' *h* as radius describe an arc cutting X X in H, and join H S.

Next draw separately B' B *h g'* (Fig. 88) one of the B' B *h g'* portions of Fig. 84, thus. Draw any line X X and

FIG. 88.

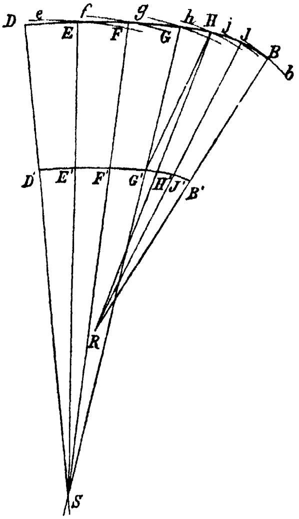


with any point P (to correspond with P, Fig. 84) in it as centre and P B (Fig. 84) as radius describe an arc B *h* equal to B *h* of Fig. 84. Make B P' equal to B P' (Fig. 84), and with P' as centre and P' B' (Fig. 84) as radius describe an arc

B′*g′* equal to B′*g′* of Fig. 84. Joining *hg′* completes the portion of Fig. 84 required. Now from B′ and P′ draw B′B″, P′P″ perpendicular to X X and each equal to the given height of the D B D portion of the bath. Join B B″, P P″; produce them to intersect in R (§ 80, p. 158); and from R let fall R R′ perpendicular to X X. Join R′*h*, cutting arc B′*g′* in *h′*, then *h* and *h′* will be corresponding points. Divide B *h* into any number of equal parts, here two, in the point *j*. Join P″B″. With R′ as centre and R′*j* and R′*h* successively as radii describe arcs cutting X X in J and H; join these points to R by lines cutting P″B″ in J′ and H′.

Next draw (Fig. 89) a line D S equal to D S (Fig. 87).

Fig. 89.



With S as centre and S E, S F, S G, and S H (Fig. 87) successively as radii describe arcs *e*, *f*, *g*, and *h*. With D as centre

and radius equal to D *e* (Fig. 87) describe an arc cutting arc *e* in E, and with same radius and E as centre describe an arc cutting arc *f* in F. With F as centre and same radius describe an arc cutting arc *g* in G, and with G as centre and radius *g h* (Fig. 87) describe an arc cutting arc *h* in H. Join the points E, F, and G (not H) to S. Make S D′ equal to S D″ (Fig. 87); and make S E′, S F′, and S G′ respectively equal to S E′, S F′, and S G′ (Fig. 87). Through the points D, E, F, G, H draw an unbroken curved line. Also through points D′, E′, F′, G′ draw an unbroken curved line, and join H G′. This completes the side pattern, to which we have now to attach, at H G′, a half-front pattern. With H and G′ as centres and radii respectively equal to H H′ and *g′ h′* (Fig. 88) describe arcs intersecting in H′. Join H H′; produce it indefinitely and make H R equal to H R (Fig. 88). With R as centre and R J, R B successively as radii describe arcs *j* and *b*. With H as centre and *h j* (Fig. 88) as radius describe an arc cutting arc *j* in J, and with same radius and J as centre describe an arc cutting arc *b* in B. Join the points J and B to R, and make R J′ and R B′ respectively equal to R J′ and R B″ (Fig. 88). Through H, J, and B draw an unbroken curved line. Also through H′, J′, and B′ draw an unbroken curved line. Then D F H B B′ G′ D′ is the complete pattern required.

PROBLEM XXIX.

To draw, **without long radii,** *the pattern for an Oxford hip-bath; given dimensions as before.*

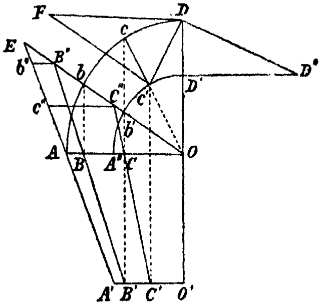
This problem is a second case of the preceding.

Patterns when the body is to be made up of three pieces with seams as in the preceding problem.

To draw the pattern for the back.

Draw E A A′ O′ O (Fig. 90) the elevation of the back Fig. 85, and produce O′ O. With O as centre and [illegible] radius describe a quadrant A D (corresponding with A D, Fig. 84), and divide it into any number of equal parts, here three, in the points b and c. Join b O, c O, and with O as centre and radius O′ A′ describe a quadrant D′ A″ (corresponding with D′ A′, Fig. 84), and cutting lines O c, O b in c' and b' respectively. Then D D′ A″ A will be the plan of that portion of the back of the bath of which O′ A′ O A is the elevation. Through b and c draw b B and c C perpendicular to O A; and through b' and c' draw b' B′ and c' C′ perpendicular to O′ A′. (Here part of b' B′ happens to coincide with part of c C). Join B′ B, C′ C, and produce them to meet O E in B″ and C″ respectively. Through C″ draw C″ c'' parallel to O A, and through B″ draw B″ b'' parallel to O A. Through D′ draw D′ D″ perpendicular to D′ D and equal to the given height of that portion of the bath, and join D D″, then D D″ will be the true length of D′ D. Join D c'; through c' draw c' F perpendicular to c' D and equal to the given height, and join D F, then D F may be taken as the true length of D c'.

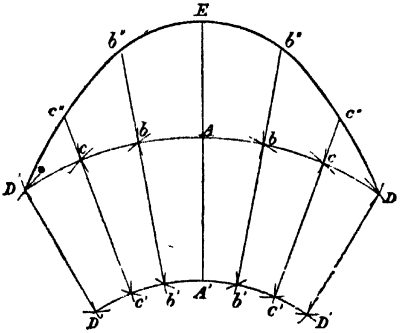
FIG. 90.



In drawing the pattern, we will first set out that for the WitO′ A′ portion of the back, and then attach to it the sively for the O E A portion. It is evident that the O A O′ A′

portion is half a right cone frustum, and therefore its pattern can be drawn (see Problem VIII., p. 41), thus. Draw (Fig. 91) a line D D′ (the line D D′ left of E A′) equal to D D″ (Fig. 90). With D and D′ as centres and radii respectively equal to D F and D′*c*′ (Fig. 90) describe arcs intersecting in *c*′. With D′ and D as centres and radii respectively equal to D F and D *c* (Fig. 90) describe arcs intersecting in *c*. To find points *b* and *b*′ proceed as just explained and with the same radii, but *c* and *c*′ as centres instead of D and D′.

FIG. 91.

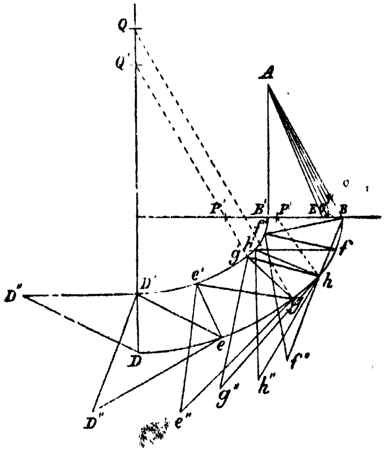


Similarly to find A and A′, and the points *b* and *b*′, &c., on the right-hand side of A A′. Through the points D′, *c*′, *b*′, A′, *b*′, *c*′, D′ draw an unbroken curved line. Also through D, *c*, *b*, A, *b*, *c*, D draw an unbroken curved line, and join D D′. This completes the pattern for the O A O′ A′ portion of back of bath, to which we have now to attach, at D A D, the pattern for the O E A portion. Join A′ A; produce it indefinitely and make A E equal to A E (Fig. 90). Next join, right and left of A A′, *b*′ *b*, *c*′ *c*, and produce them indefinitely; make *b* *b*″, right and left of A A′, equal to A *b*″ (Fig. 90). Also make *c* *c*″, right and left of A A′, equal to A *c*″ (Fig. 90), and through the points D, c'', b'', E, b'', c'', D draw an unbroken curved line; then D E D D' A' D' is the complete pattern required.

To draw the side and half-front pattern.

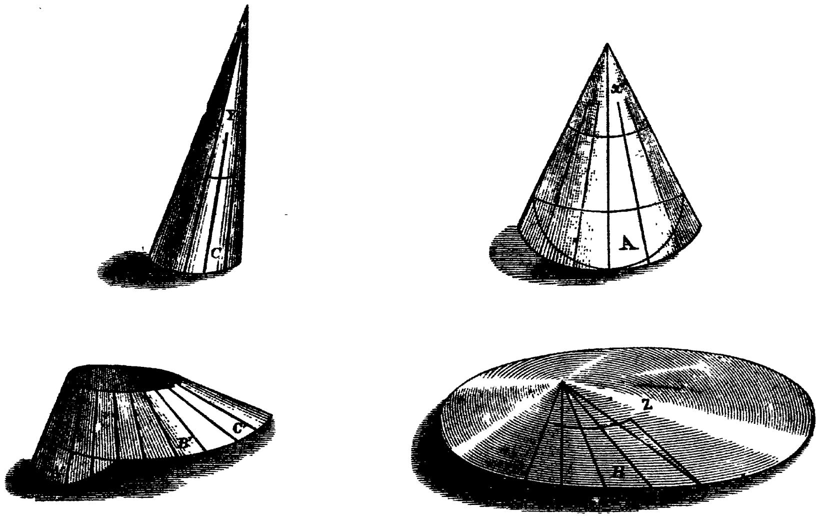
Draw separately (Fig. 92) the B *h* D D' *g'* B' portion of Fig. 84, that is, the plan, of a side and half-front of the bath Join *h g'* (as was done in Fig. 84). We now have to get at corresponding points in the arcs D *h*, D' *g'*, and also in the arcs *h* B, *g'* B. We do this by the method given in § 85, p. 202,

FIG. 92.



thus. Join Q' *g'* and through Q draw Q *g* parallel to Q' *g'* and cutting arc D *h* in *g*; then the arcs D *g* and D' *g'* will be the proportional. Now divide arc D *g* into any number of equal parts, here two only to avoid confusion, in the point *e*, and divide the arc D' *g'* into two equal parts in the point *e'*; then *e*, *e'* and *g*, *g'* are corresponding points. Next join P *h* and through P' draw P' *h'* parallel to P *h* and cutting arc B' *g'* in *h'*.

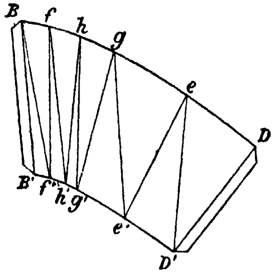
PLATE IV. (see p. 217).



AWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (*Part of Exhibit by the Author at the Inventions Exhibition, 1885.*)

parts, here two, in the points f and f' respectively; then f, f' and h, h' are corresponding points. Join $e\,e'$, $g\,g'$, $f\,f'$, and $h\,h'$. Through B' draw $B'\,A$ perpendicular to $B'\,B$ and equal to the given height of that portion of the bath; and from B' along $B'\,B$ mark off $B'\,F$, $B'\,H$, $B'\,G$, and $B'\,E$ respectively equal to $f\,f'$, $h\,h'$, $g\,g'$, and $e\,e'$. Join B A, F A, H A, G A and E A; then B A, F A, H A, G A, and E A will be respectively the true lengths of $B\,B'$, $f\,f'$, $h\,h'$, $g\,g'$, and $e\,e'$. Join $f'\,B$, $h'\,f$, $e'\,g$, and $D'\,e$; through f' draw $f'\,f''$ perpendicular to $f'\,B$, and equal to the given height, and join $B\,f''$, then $B\,f''$ may be taken as the true length of $f'\,B$. Through h', g', e', and D' draw $h'\,h''$, $g'\,g''$, $e'\,e''$, and $D'\,D''$, perpendicular to $h'\,f$, $g'\,h$, $e'\,g$, and $D'\,e$ respectively, and each equal to the given height; also draw $D'\,D''$ perpendicular to $D\,D'$ and equal to the given height; and join $f\,h''$, $h\,g''$ $g\,e''$, $e\,D''$, and $D\,D''$; then $f\,h''$, $h\,g''$, $g\,e''$, $e\,D''$, and $D\,D''$ may be taken as the true lengths of $h'\,f$, $g'\,h$, $e'\,g$, $D'\,e$, and $D'\,D$ respectively.

FIG. 93.



Next draw (Fig. 93) $B\,B'$ equal to B A (Fig. 92), and with B and B' as centres and radii respectively equal to $B\,f''$ and $B'\,f'$ (Fig. 92) describe arcs intersecting in f', and with f' and B as centres and radii respectively equal to F A and $B\,f$ (Fig. 92) describe arcs intersecting in f. With f and f' as [illegible] and radii respectively equal to $f\,h''$ and $f'\,h'$ (Fig. 92) describe arcs intersecting in h', and with h' and f as centres and radii respectively equal to H A and fh (Fig. 92) describe arcs intersecting in h. With h and h' as centres and radii respectively equal to $h\,g''$ and $h'\,g'$ (Fig. 92) describe arcs intersecting in g', and with g' and h as centres and radii respectively equal to G A and $h\,g$ (Fig. 92) describe arcs intersecting in g. With g and g' as centres and radii respectively equal to $g\,e''$ and $g'\,e'$ (Fig. 92) describe arcs intersecting in e', and with e' and g as centres and radii respectively to E A and $g\,e$ (Fig. 92) describe arcs intersecting in e. With e and e' as centres and radii respectively equal to e D″ and e' D′ (Fig. 92) describe arcs intersecting in D′, and similarly with D′ and e as centres and radii respectively equal to D D″ and e D (Fig. 92) describe arcs intersecting in D. Join D D′. Through B, f, h, g, e, D draw an unbroken curved line. Also through B′, f', h', g', e', D′ draw an unbroken curved line. Then B g D D′ g' B′ is the pattern required.

PROBLEM XXX.

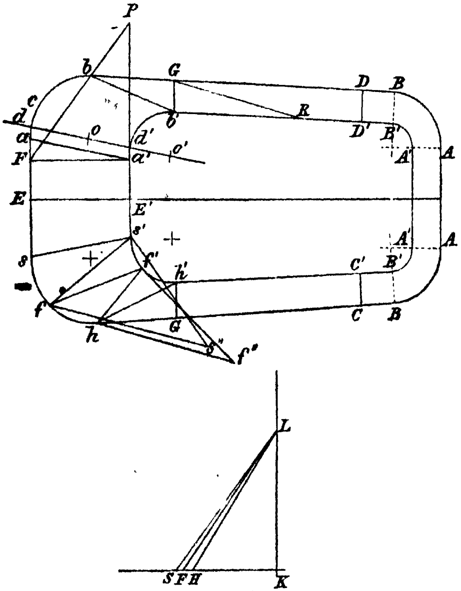
To draw the pattern for an oblong taper bath, the like dimensions to those for Problem XIII. being given.

Again, it is only necessary to treat of two cases—one in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 94) the plan of the body (see Problem XIII., p. 140), preserving of its construction the centres O, O′; and the points b, b', a, a', s, s', h, h', A, A′, B, B′, in which the straight lines and arcs meet each other. Join b, b', a, a', s, s', h, h', A A′ (two places), and B B′ (two places) as shown in the fig. Examining the plan we see (d, p. 55) that each round corner A A′ B B′ of the toe is the same portion of a right cone frustum; and each of the round corners $a\,a'\,b'\,b$, $s\,s'\,h'\,h$, of the head are the same portion (g, p. 129) of an oblique cone frustum. As we proceed it will be seen the construction of the pattern for the round corners of the head of bath is exactly the same as that for the round corners of the body in Problem XXVI. (see also § 86, p. 206).

In Plate V. (p. 237) is a representation of an oblong taper bath, also of an oblique cone Z, the A portion of which corresponds to the A′ portion of the body, and the development of the former is the development of the latter.

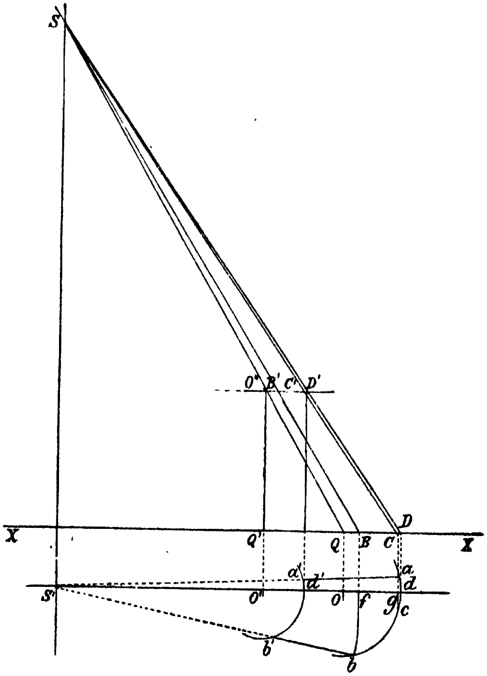
FIG. 94.



Patterns when the body is to be made up of four pieces.

We will put the seams to correspond with the lines G h′, G b′, D D′, and C C′. The patterns required will be three, one for the head of the bath, one for the toe, and one for the sides. The pattern for the toe can be readily drawn by Problem XXVII, p. 90. Likewise the pattern for the sides. The pattern for the head is drawn as follows.

FIG. 95

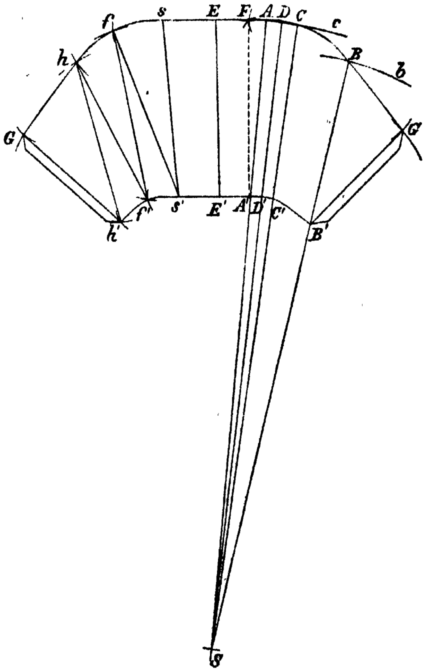


Draw separately (Fig. 95) a head-corner portion of Fig. 94, say *a a' b' b*, thus. Draw an indefinite line S'*d* (Fig. 95), and with any point O (corresponding to O, Fig. 94) in it as centre and O*a* (Fig. 94) as radius describe an arc *b a*. Join O O' (Fig. 94) and produce it to cut *b a* in *d*; make *d b* and *d a* (Fig. 95) equal respectively to *d b* and *d a* (Fig. 94). Now (Fig. 95) make *d* O′ equal to *d* O′ (Fig. 94), and with O′ as centre and O′*a*′ (Fig. 94) as radius describe an arc *b*′*a*′. Make *d*′*b*′ and *d*′*a*′ equal respectively to *d*′*b*′ and *d*′*a*′ (Fig 94). Joining *b b*′, *a a*′ completes the portion of Fig. 94 required. Now divide (Fig. 94) *b a* into any number of parts. It is convenient to take *d* as one of the division points, and to make *d c* equal to *d a*; leaving *c b* without further division, thus making the division of *b a* into three portions, not all equal. In actual practice the dimensions of the work will suggest the number of parts necessary. Here *b c* is left without further division in order to make clear the correspondence of this problem to Problem XXVI., p. 205. Now (Fig. 95) make *d c* equal to *d c* (Fig. 94), and then *b a* will be divided correspondingly to *b a* (Fig. 94). Draw X X parallel to S′*d*; and at *d* and O draw *d* D, O Q perpendicular to S′*d*; and meeting X X in D and Q; also through *d*′ and O′ draw *d*′ D′, O′O″ perpendicular to S′*d*; the line O′O″ cutting X X in Q′. Make Q′O″ equal to the given height of the bath, and draw O″D′ parallel to X X, and cutting *d*′ D′ in D′. Join D D′, Q O″; produce them to intersect in S (§ 80, p. 158); and from S let fall a perpendicular to S′*d*, cutting S′*d*′ in S′. With S′ as centre and S′*a*, or S′*c* (which is equal to S′*a*) and S′*b* successively as radii describe arcs cutting S′*d* in *g* and *f*. Draw *g* C, *f* B perpendicular to X X and cutting it in C and B; and join C S, B S, cutting O″D′ in C′ and B′.

Next draw S D (Fig. 96) equal to S D (Fig. 95) and with S as centre and S C, S B (Fig. 95) successively as radii describe arcs *c* and *b*. With D as centre and radius equal to *d a* (Fig. 95) describe arcs cutting arc *c* in A and C. With C as centre and radius *c b* (Fig. 95) describe an arc cutting arc *b* in B. Join A, C, and B to S. Make S D′ equal S D′ (Fig. 95) and with S as centre and S C′ (Fig. 95) describe an arc (not shown in the fig.) cutting S A and S C in A and C respectively; make S B′ equal to S B′ (Fig. 95).

Through A, D, C, and B draw an unbroken curved line. Also through A′, D′, C′, and B′ draw an unbroken curved line; this will complete the pattern for a head-corner. To attach the patterns for the flat portions of the head to A A′

Fig. 96.



and B B′ respectively; draw through a' (Fig. 94) a line a' F perpendicular to a' s', and through b' draw b' G perpendicular to b' D′.

Now draw (Fig. 94) a' P perpendicular to a' F′ and equal to the given height of the bath, and join F P, then F P is the true length of F *a'*.

Next draw *b'* R perpendicular to *b'* G (*b'* R will of course coincide with the line *b'* B') and equal to the given height; join G R, then G R is the true length of G *b'*. Now with B' (Fig. 96) as centre and G R (Fig. 94) as radius, and B as centre and radius *b* G (Fig. 91), describe arcs intersecting in G. Join B G, B' G. With A' (Fig. 96) as centre and F P (Fig. 94) as radius, and A as centre and radius *a* F (Fig. 94) describe arcs intersecting in F. Join A F and produce it indefinitely, and make A *s* equal to *a s* (Fig. 94); through A' draw A' *s'* parallel to A *s* and equal to *a' s'* (Fig. 94); and join *s s'*. The pattern for the portion, seen in plan in Fig. 94, G *b a s s' a' b'* of the head of the bath is now completed. It is needless to work out in detail the addition of the portion (Fig. 96) *s h* G *h' f' s'* of the pattern, by which we complete the head pattern G E G *h'* E' B'. The extra lines in this latter portion of the pattern appertain to the next problem.

PROBLEM XXXI.

To draw, **without long radii**, *the pattern for an oblong taper bath; given dimensions as in Problem XXX.*

This problem is a second case of the preceding.

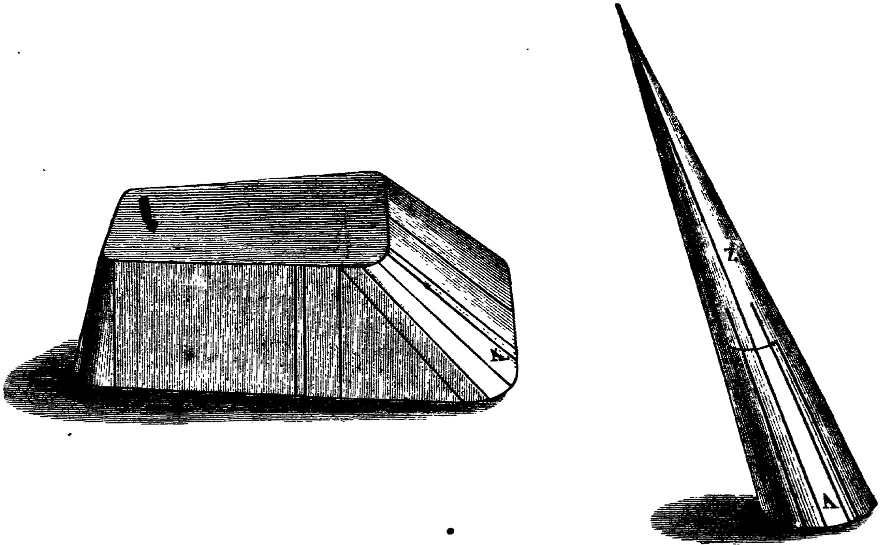
Patterns when the body is to be made up of four pieces with seams as in preceding problem.

Again, the patterns required will be three; one for the head of the bath, one for the toe, and one for the sides. The latter pattern needs no description. The pattern for the toe can be readily drawn by Problem XXVIII., p. 94. The pattern for the head can be drawn as follows.

Draw half the plan of the bath, as the lower half of Fig. 94, and divide the arcs *s h*, *s' h'*, each into any number of equal parts, here two, in respectively the points *f* and *f'*, and join *f f'*. Draw (Fig. 94*a*) any two lines K S, K L perpendicular to each other and make K L equal to the given height of the bath. From K along K S mark off K H equal to *h h'*, K F equal to *f f'*, and K S equal to *s s'*; and join L H, L F, L S; then L H, L F, and L S are the true lengths of *h h'*, *f f'*, and *s s'* respectively. Next join (Fig. 94) *f' h*, *s' f*; draw *f' f''*, *s' s''*, perpendicular to *f' h*, *s' f* respectively, and each equal to the given height; and join *h f''*, *f s''*; then *h f''*, *f s''* may be taken respectively as the true lengths of *f' h* and *s' f*. The true length of *h'* G may be found along *h'* B' as was that of *b* G in Problem XXX. along *b'* B'; it will of course be equal to G R, and we shall speak of it as G R.

Next draw (see Fig. 96, left-hand portion) *s s'* equal to L S (Fig. 94*a*), and with *s'* and *s* as centres and radii respectively equal to *f s''* and *s f* (Fig. 94) describe arcs intersecting in *f*. With *f* and *s'* as centres and radii respectively equal to L F (Fig. 94*a*) and *s' f'* (Fig. 94) describe arcs intersecting in *f'*; and with *f'* and *f* as centres and radii respectively equal to *h f''* and *f h* (Fig. 94) describe arcs intersecting in *h*; also with *h* and *f'* as centres and radii respectively equal to L H (Fig. 94*a*) and *f' h'* (Fig. 94) describe arcs intersecting in *h'*. With *h'* and *h* as centres and radii respectively equal to G R and *h* G (Fig. 94) describe arcs intersecting in G. Through *s*, *f*, *h*, draw an unbroken curved line. Also through *s'*, *f'*, *h'* draw an unbroken curved line; and join *h* G, G *h'*. Then *s h* G *h' f' s'* is the pattern for the portion of the head of the bath represented in plan in Fig. 94 by the same lettering. It is unnecessary to pursue the pattern further.

PLATE IV. (see p. 251.)



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (*Part of Exhibit by the Author at the Inventions Exhibition,* 1885.)